Do Communication Engineers Need Circuit Theory?

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Taipei, Taiwan

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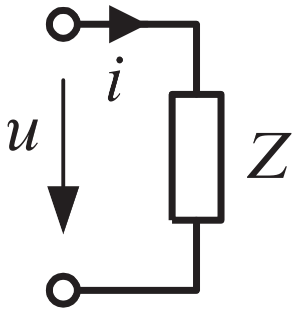
Contents

How to Compute Power Flow

one-port



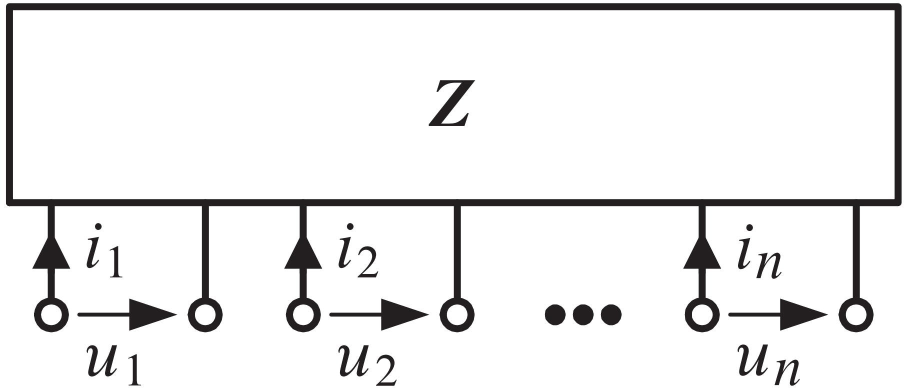
u, i complex phasors

$$P = \mathrm{Re}\{u^* i\}$$

$$= |i|^2 \, \mathrm{Re}\{Z\}$$

$$= |u|^2 \, \mathrm{Re}\{Z^{-1}\}$$

reciprocal multi-port



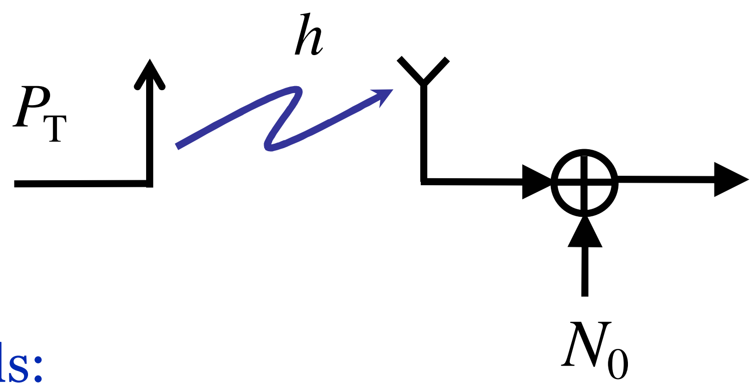
$$P = \sum_{k=1}^{n} \mathrm{Re}\{u_k^* i_k\} = \mathrm{Re}\{\boldsymbol{u}^{\mathrm{H}} \boldsymbol{i}\}$$

$$= \boldsymbol{i}^{\mathrm{H}} \mathrm{Re}\{\boldsymbol{Z}\} \boldsymbol{i}$$

$$= \boldsymbol{u}^{\mathrm{H}} \mathrm{Re}\{\boldsymbol{Z}^{-1}\} \boldsymbol{u}$$

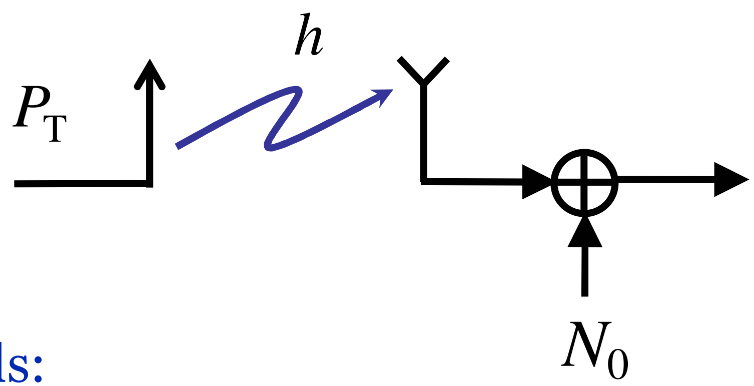
Contents

Single Antenna Systems and their Limitations



Symbols:

- P_T : Transmit Power
- h : (complex) channel gain
- N_0 : (two-sided) noise power density
- T : Temporal Channel-use Spacing
- B : Noise Bandwidth
- B_S : Occupied Signal Bandwidth



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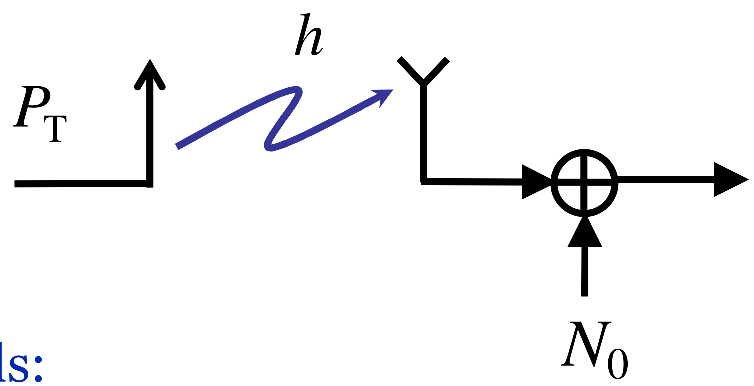
Signal to Noise Ratio

$$\text{SNR} = \frac{|h|^2 P_T}{B \cdot N_0}$$



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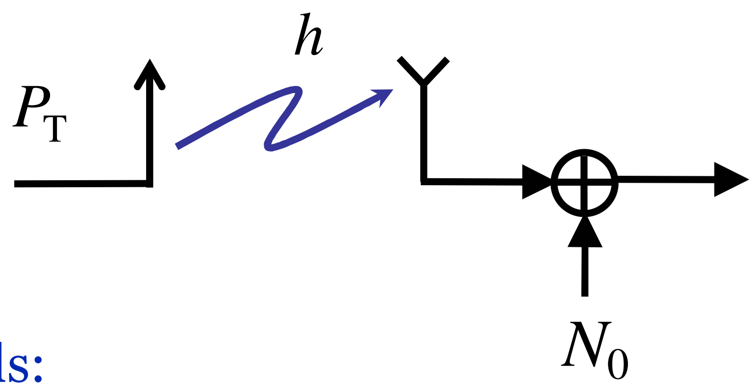
Signal to Noise Ratio

$$\mathrm{SNR} = \frac{|h|^2 P_T}{B \cdot N_0}$$

Channel-use Spacing

$$T = \frac{1}{B}$$

for square-root Nyquist pulse shape



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$$B_S = (1 + \rho) B$$

roll-off factor ρ

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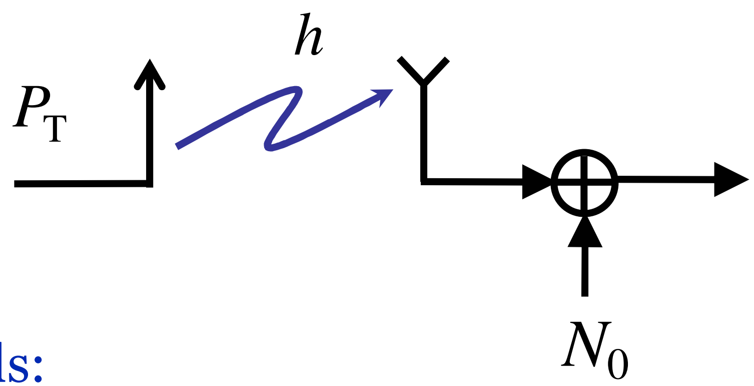

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Occupied Bandwidth

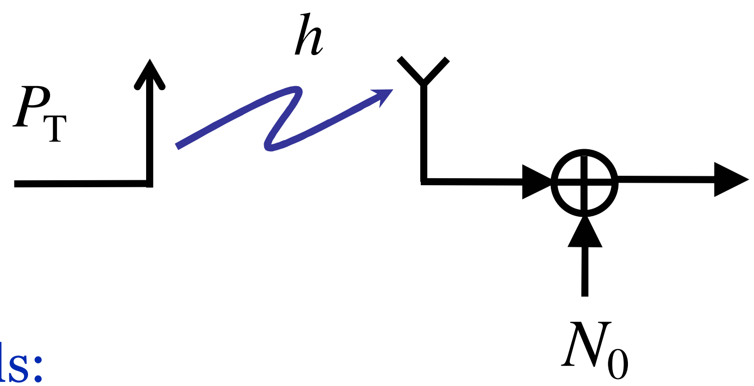
$$B_S = (1 + \rho)B$$ roll-off factor ρ

Channel Capacity

$$C = \log_2(1 + \mathrm{SNR}) \quad \frac{\text{bits}}{\text{channel-use}}$$

$$\widetilde{C} = B \log_2(1 + \mathrm{SNR}) \quad \frac{\text{bits}}{\text{second}}$$

Single Antenna Systems and their Limitations



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$$\tilde{C}_{max} = \lim_{B \to \infty} B \log_2 \left(1 + \frac{|h|^2 P_T}{B \cdot N_0} \right) = \frac{|h|^2 P_T}{N_0 \ln 2}$$

Signal to Noise Ratio

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Bandwidth Efficiency

$$\eta_{\mathrm{B}} = \frac{\widetilde{C}}{B_{\mathrm{S}}} \overset{\rho=0}{=} \frac{\widetilde{C}}{B} = \log_2(1 + \mathrm{SNR})$$

Interpretation:

Information at rate of η_{B} bits/second can be transfered in each Hz of bandwidth.

$$0 \leq \eta_{\mathrm{B}} < \infty$$

The larger η_{B}, the better we utilize bandwidth!

Transmit Power Efficiency

$$\eta_{\mathrm{P}} = \frac{C}{\mathrm{SNR}} = \frac{\widetilde{C} \cdot N_0}{|h|^2 P_{\mathrm{T}}} = \frac{\log_2(1 + \mathrm{SNR})}{\mathrm{SNR}}$$

$$= \frac{1}{\ln 2} \cdot \frac{\widetilde{C}\,(P_{\mathrm{T}})}{\widetilde{C}_{\max}(P_{\mathrm{T}})}$$

Interpretation:

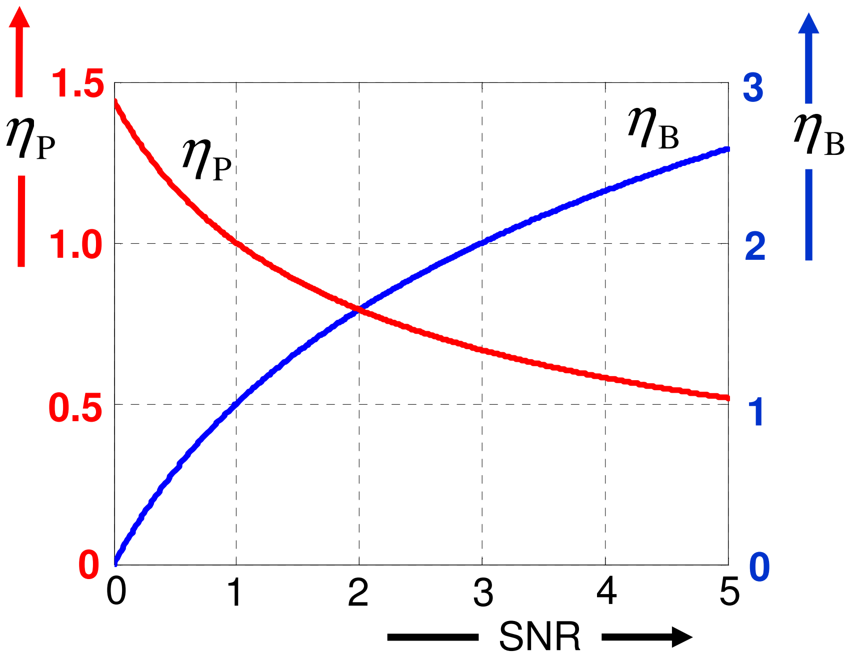
η_{P} measures the relative utilization of transmit power with respect to maximum possible channel capacity.

$$0 < \eta_{\mathrm{P}} \leq \frac{1}{\ln 2} \approx 1.44$$

The larger η_{P}, the better we utilize transmit power!

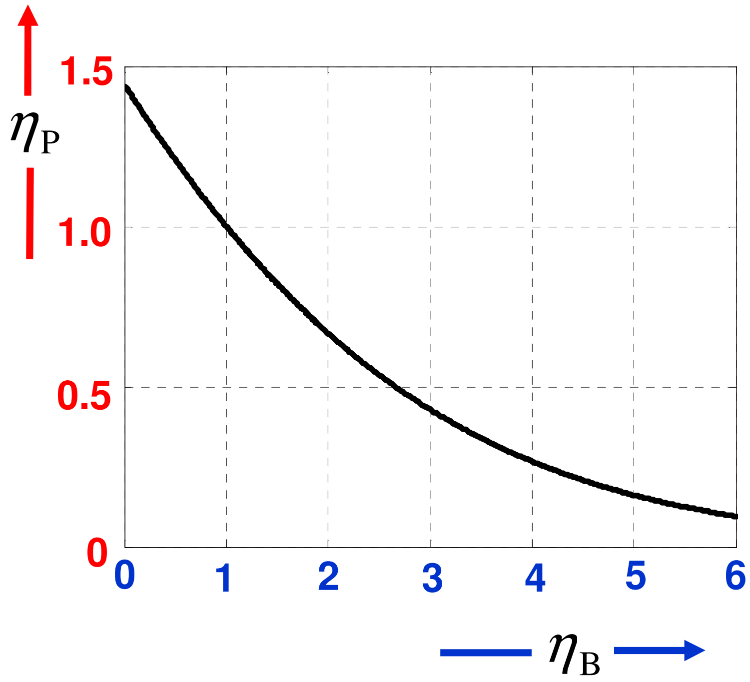
✻ Bandwidth Efficiency

$$\eta_B = \log_2(1 + \mathrm{SNR})$$



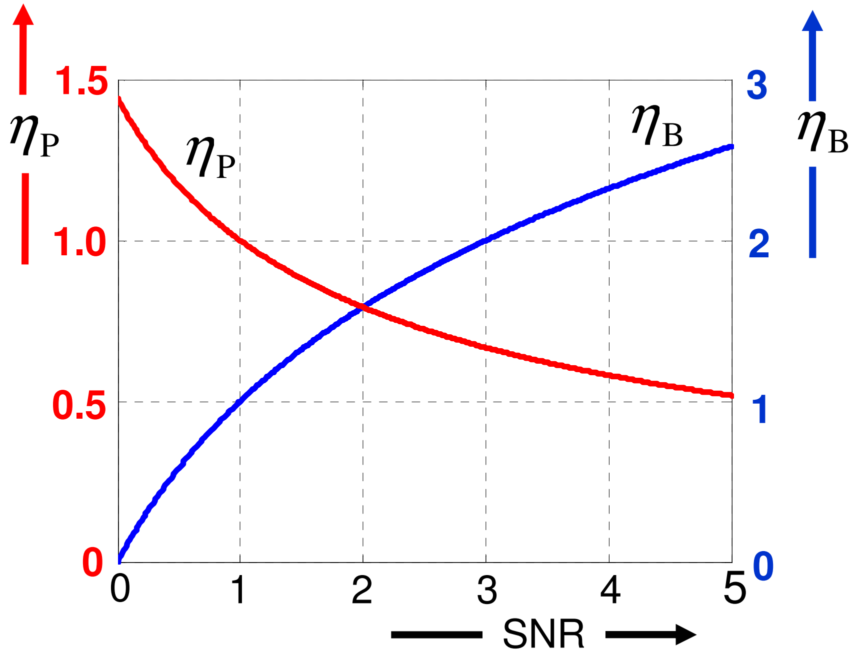
✻ Transmit Power Efficiency

$$\eta_P = \frac{\log_2(1 + \mathrm{SNR})}{\mathrm{SNR}} = \frac{\eta_B}{2^{\eta_B} - 1}$$



- **Bandwidth Efficiency**

$$\eta_B = \log_2(1 + \mathrm{SNR})$$



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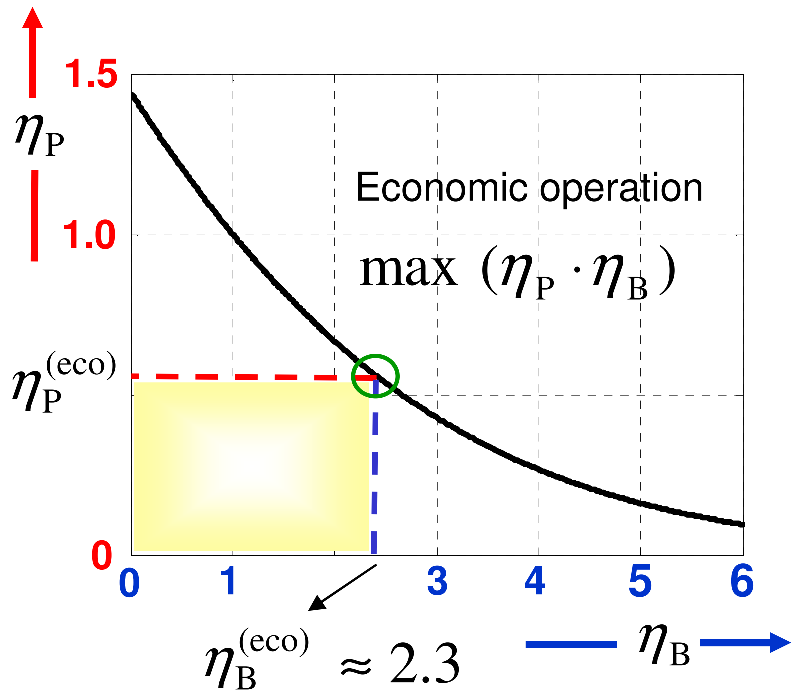
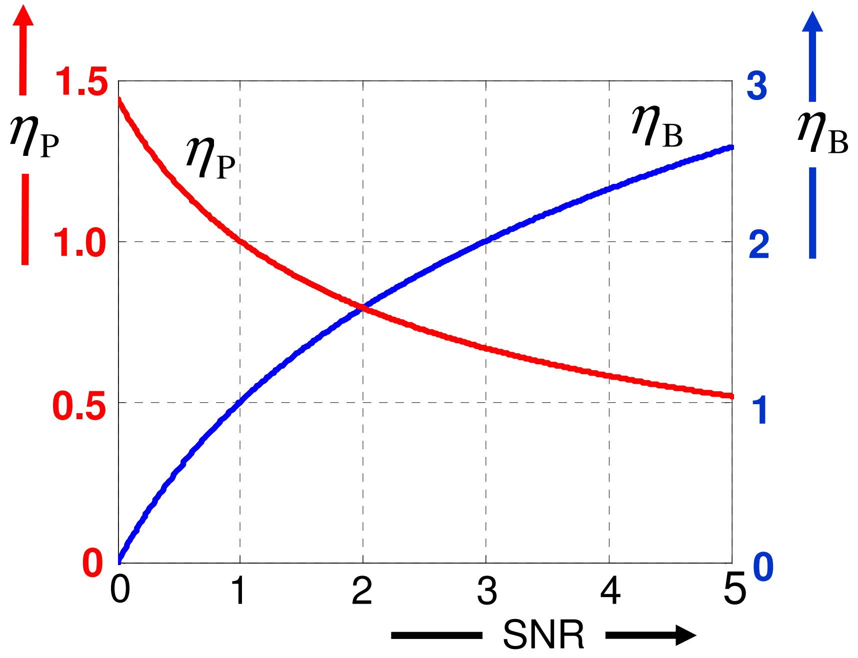


Economic Operation 3/4

- **Bandwidth Efficiency**

$$\eta_B = \log_2(1 + \mathrm{SNR})$$



- **Transmit Power Efficiency**

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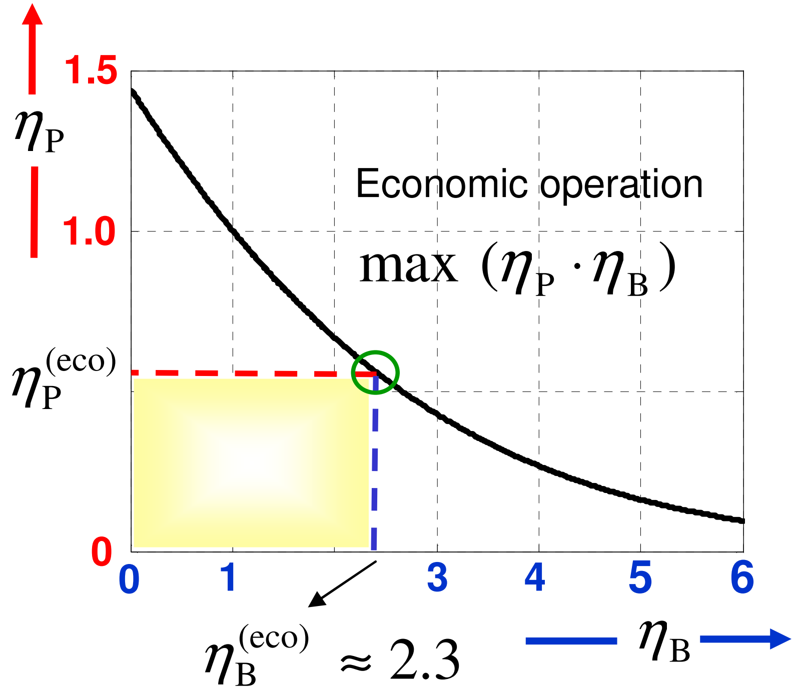
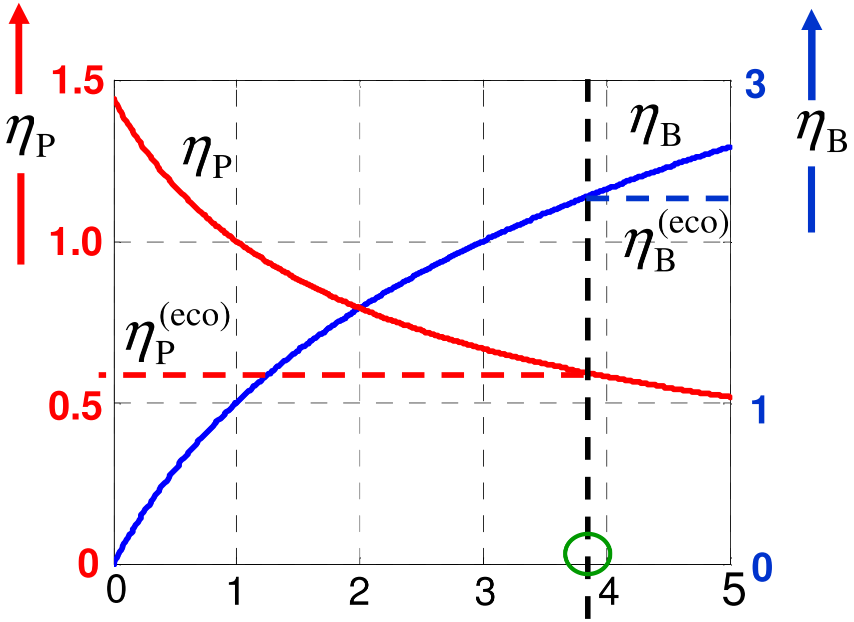


A power economic single-channel system provides an information rate of no more than 2.3 bits/second for each Hz of bandwidth.

✻ Bandwidth Efficiency

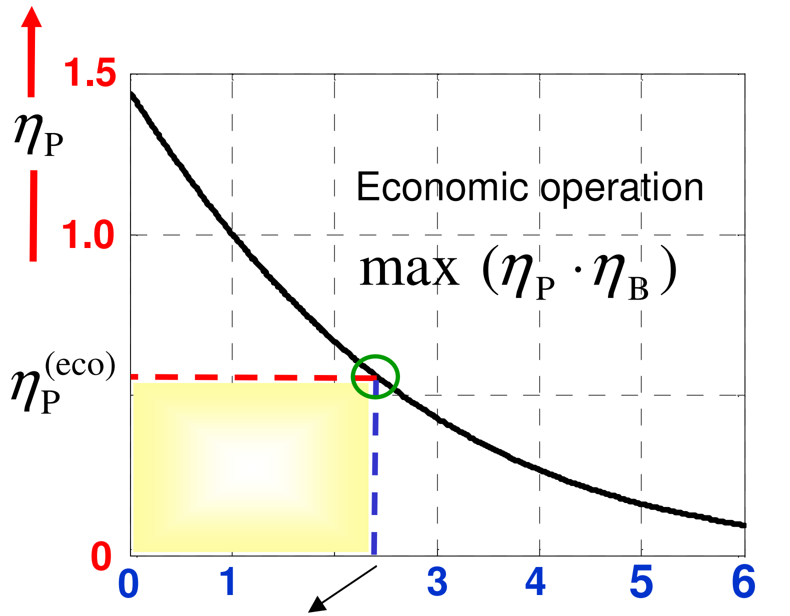
$$\eta_B = \log_2(1 + \text{SNR})$$



$\text{SNR}^{(eco)} \approx 3.92 \triangleright 6\text{dB}$

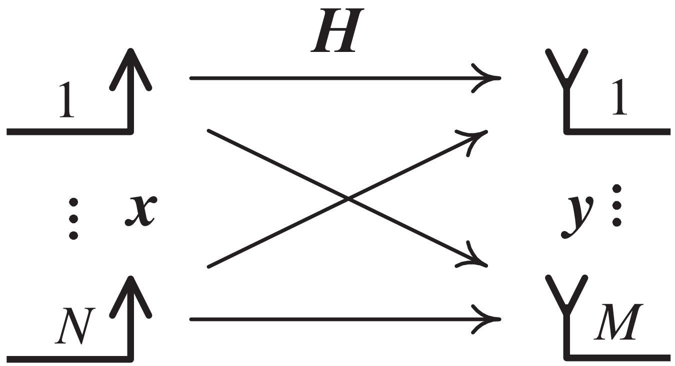
✻ Transmit Power Efficiency

$$\eta_P = \frac{\log_2(1 + \text{SNR})}{\text{SNR}} = \frac{\eta_B}{2^{\eta_B} - 1}$$



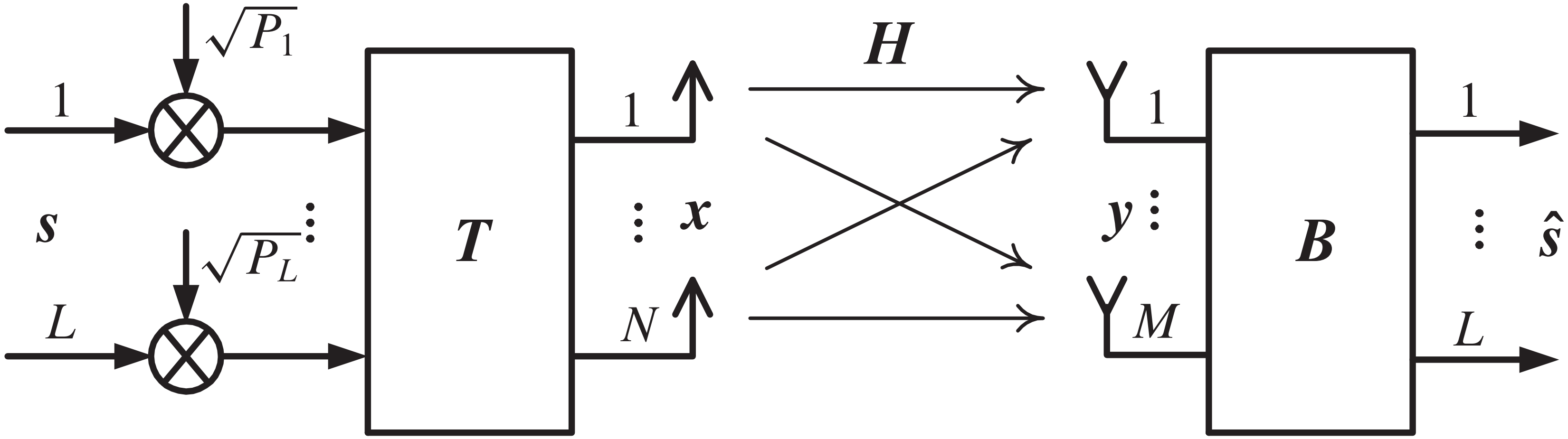
$\eta_B^{(eco)} \approx 2.3$

A power economic single-channel system has to operate at SNR of no more than 6 dB.



$$y = Hx + n$$

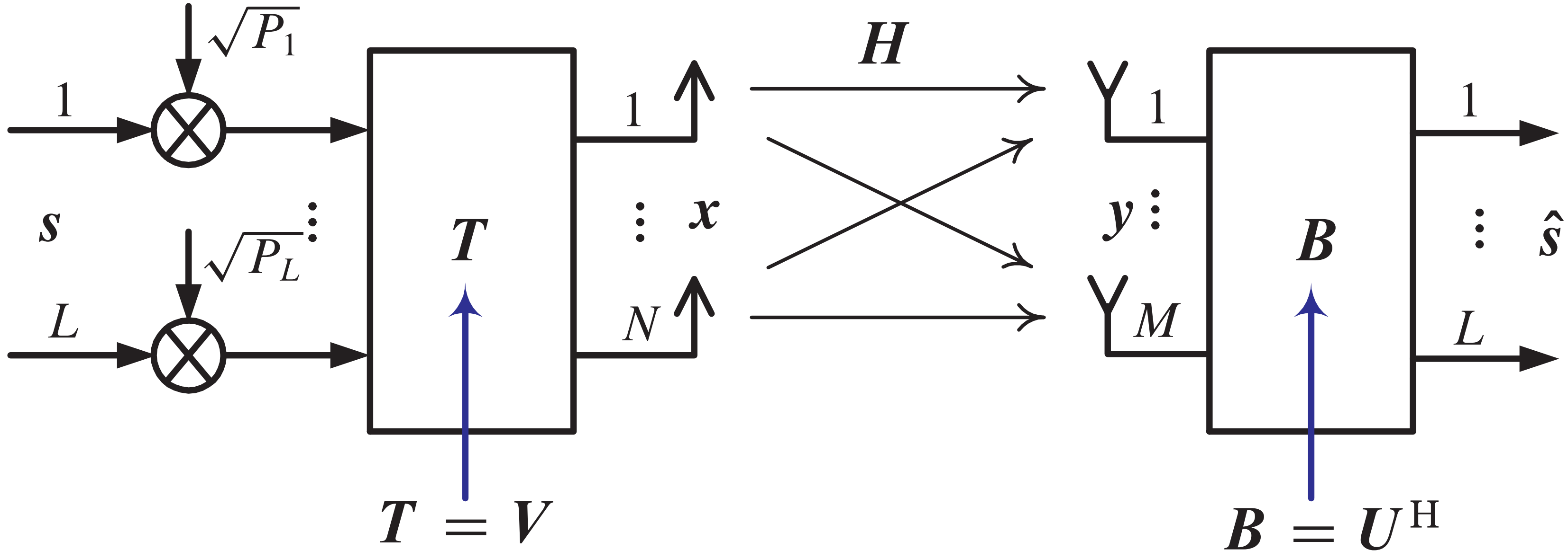
Multi-Channel Radio Link



$$\hat{\boldsymbol{s}} = \boldsymbol{B}\boldsymbol{H}\boldsymbol{T}\boldsymbol{P}^{1/2}\boldsymbol{s} + \boldsymbol{B}\boldsymbol{n}$$

$$\begin{aligned} L &= \operatorname{rank}(\boldsymbol{H}) \\ &\leq \min(M, N) \\ \boldsymbol{P} &= \operatorname{diag}\{P_i\}_{i=1}^{L} \end{aligned}$$

Multi-Channel Radio Link



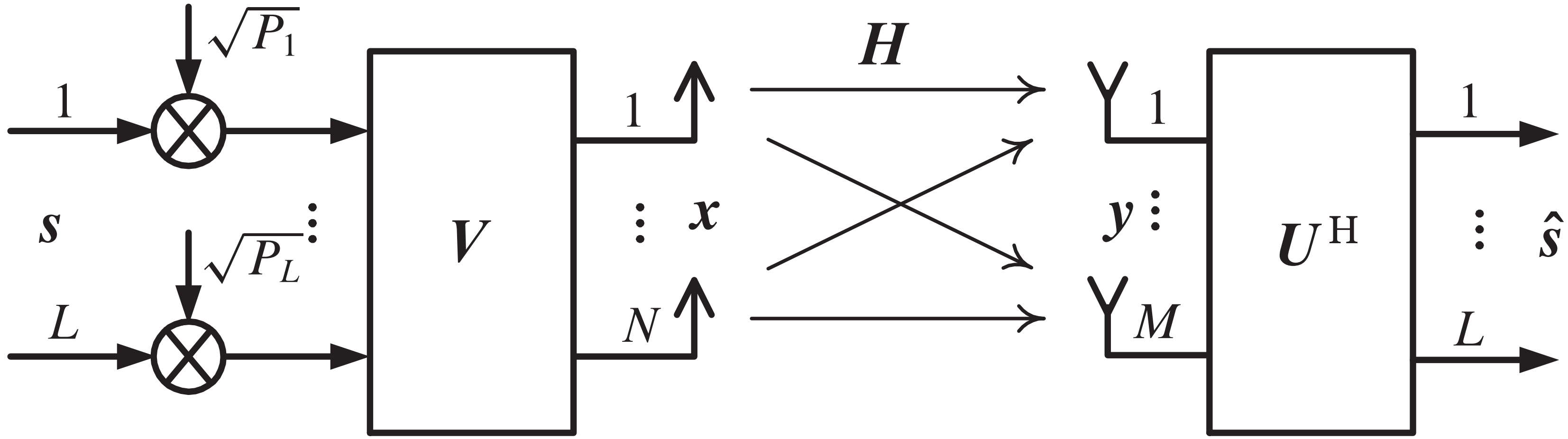
Singular Value Decomposition

$$\hat{s} = BHTP^{1/2}s + Bn$$

$$\hat{s} = BU\begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_L \end{bmatrix}V^{H}TP^{1/2}s + n'$$

$$L = \mathrm{rank}(H) \leq \min(M, N)$$

$$P = \mathrm{diag}\{P_i\}_{i=1}^{L}$$



Singular Value Decomposition

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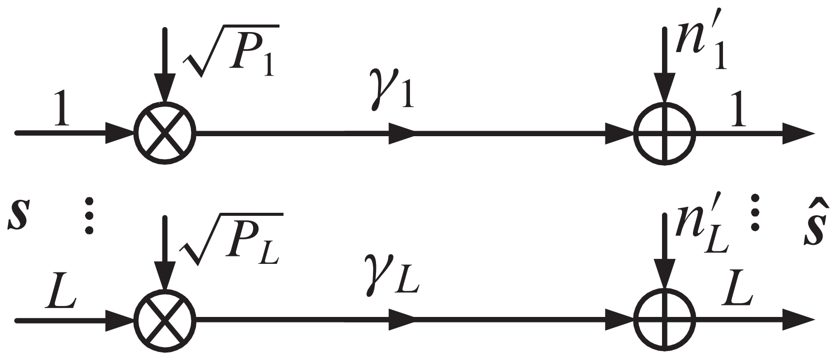
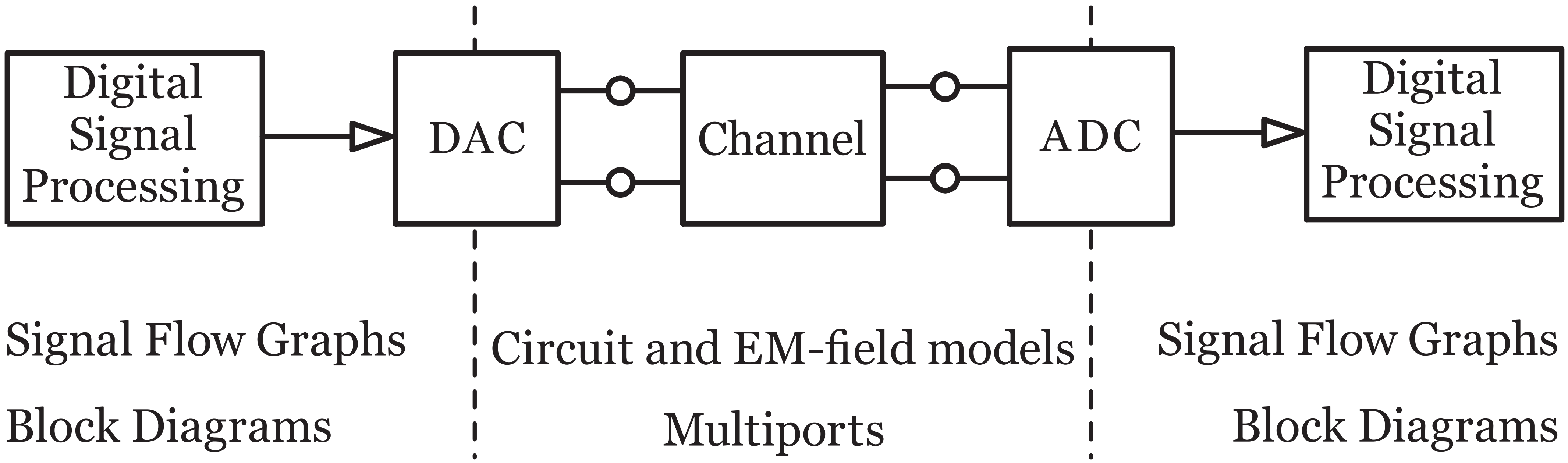


Contents

- *First: Get the physics right!*
  Second: The rest is mathematics! R. Kalman, 2005

- *Did we, the system theorists, get the physics right? Do our basic model structures adequately translate physical reality? Does the way in which we view interconnections respect the physics?* J. Willems, 2007

- Did we, the communication and information engineers, get the physics right in our models?



Power/Energy is always determined by two port variables, and in general, not by the squared magnitude of only one variable!

- Bring the physics's constraints into information theory!

  ⟶ Theory instantly becomes applicable to the specific system

- Is it practical to do that?

  ⟶ Yes, because it can be done without changing the theory at all!

- Is it fun?

  ⟶ Yes, because the system may turn out to perform *better* than expected, once we bring in the governing physics!

* Some examples of the »funny« things that can happen:

1. A uniform linear antenna array of N isotrops can achieve a transmit array gain of N^2.
2. The receive array gain of a uniform linear antenna array of isotrops can grow exponentially with the number of antennas.
3. The channel capacity can grow linearly with the number of receive antennas, even when only a single transmit antenna is used (SIMO).
4. The channel capacity of MIMO systems can grow quadratic with the number of antennas.

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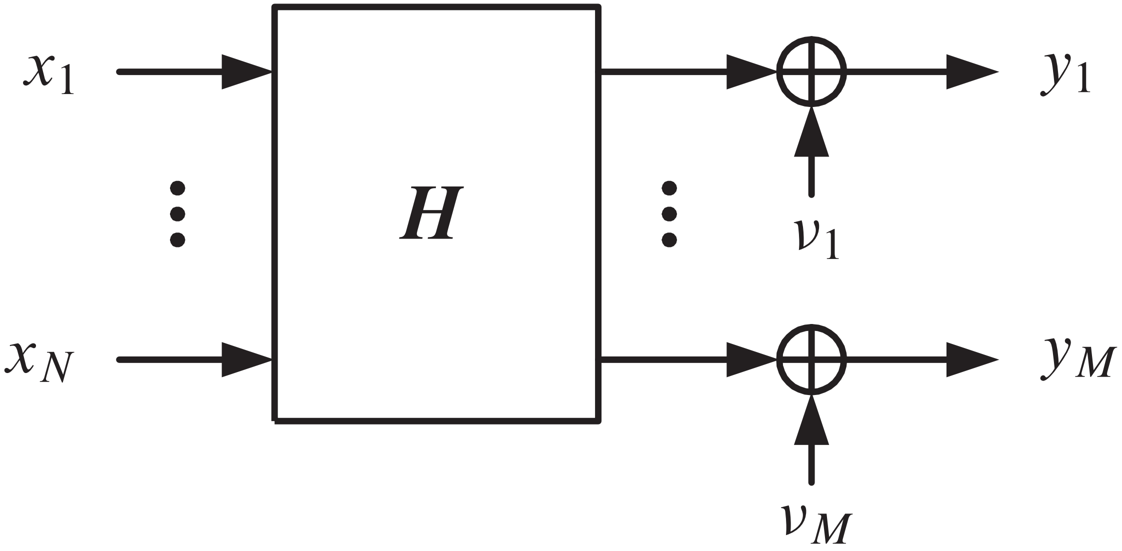
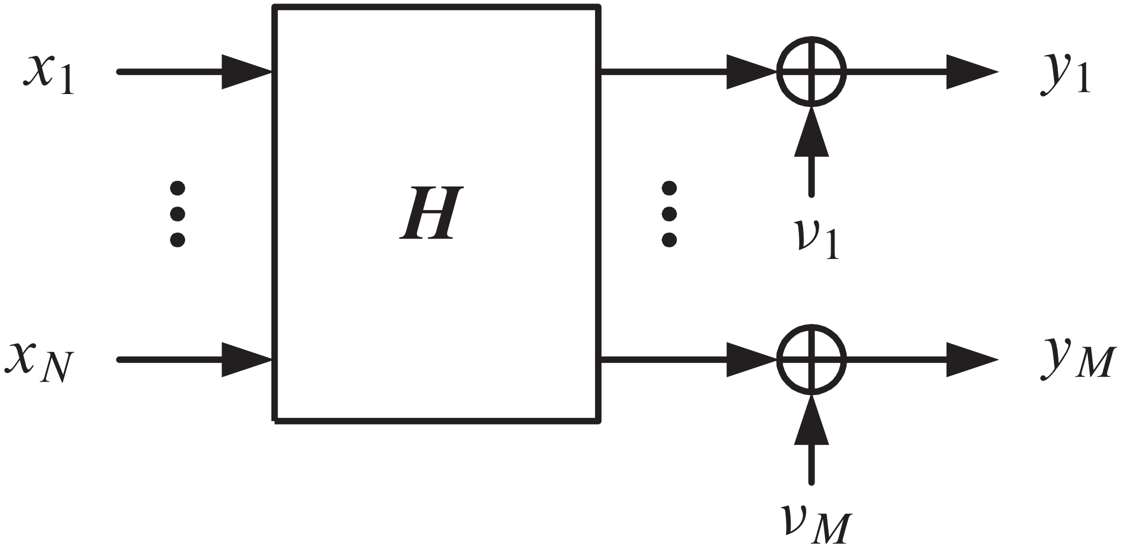
The common view on MIMO systems

Mathematical Model



- Additive noise vector channel
- N inputs, M outputs
- $\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}$
- Channel matrix: $\boldsymbol{H} \in \mathbb{C}^{M \times N}$

Mathematical Model



- Additive noise vector channel
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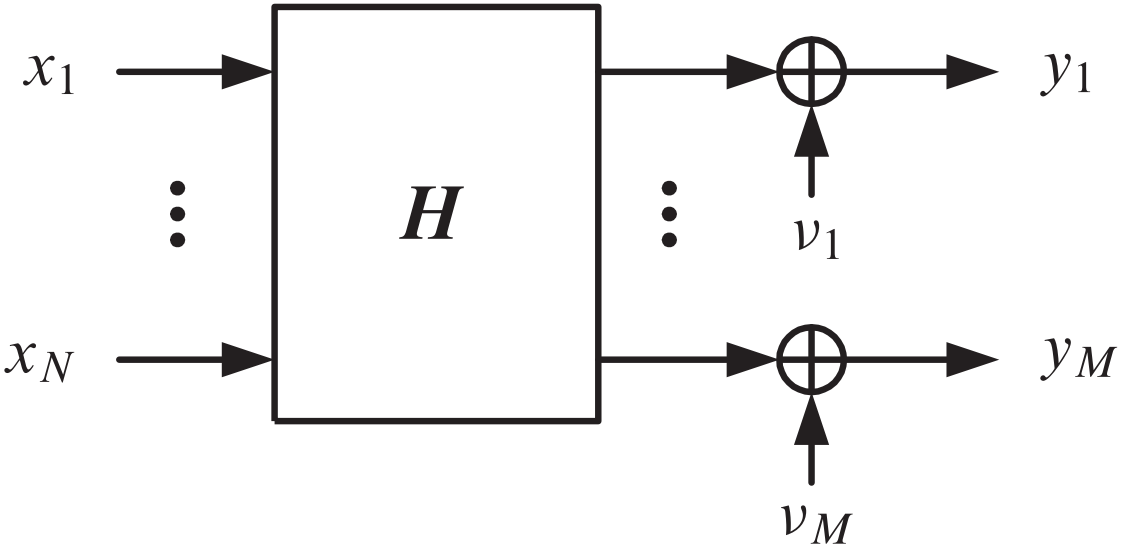
Channel Capacity

»transmit power constraint«

$$C = \max_{\text{pdf}(\boldsymbol{x})} \mathrm{I}\left(\text{pdf}\left(\boldsymbol{y}|\boldsymbol{x}\right), \text{pdf}\left(\boldsymbol{x}\right)\right), \quad \text{subject to} \quad \underbrace{\mathrm{E}\left[||\boldsymbol{x}||_2^2\right] \leq P_{\text{Tx}}}$$

✻ Mathematical Model



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✻ A question: How is $\mathrm{E}\left[||\boldsymbol{x}||_2^2\right]$ related to physical transmit power?

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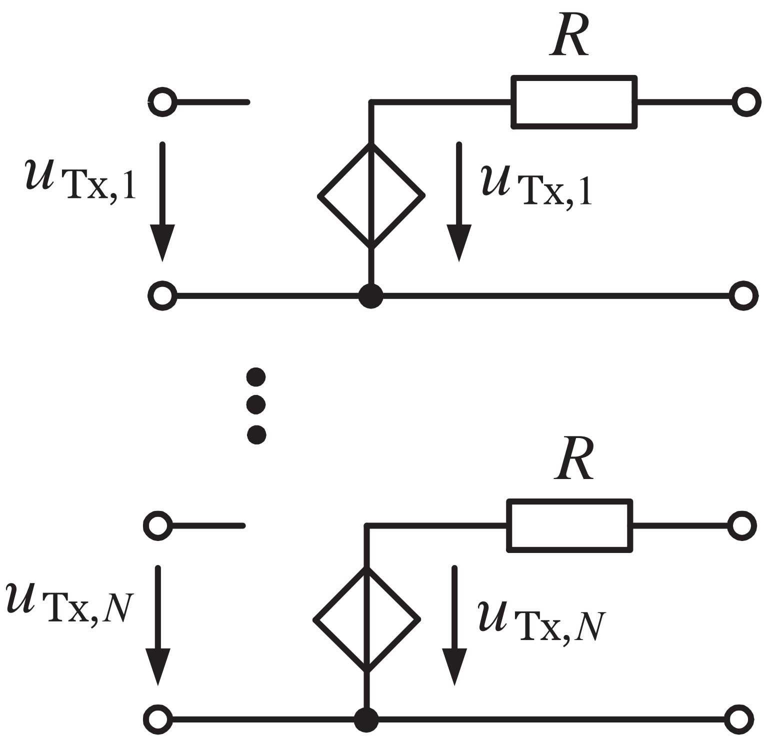
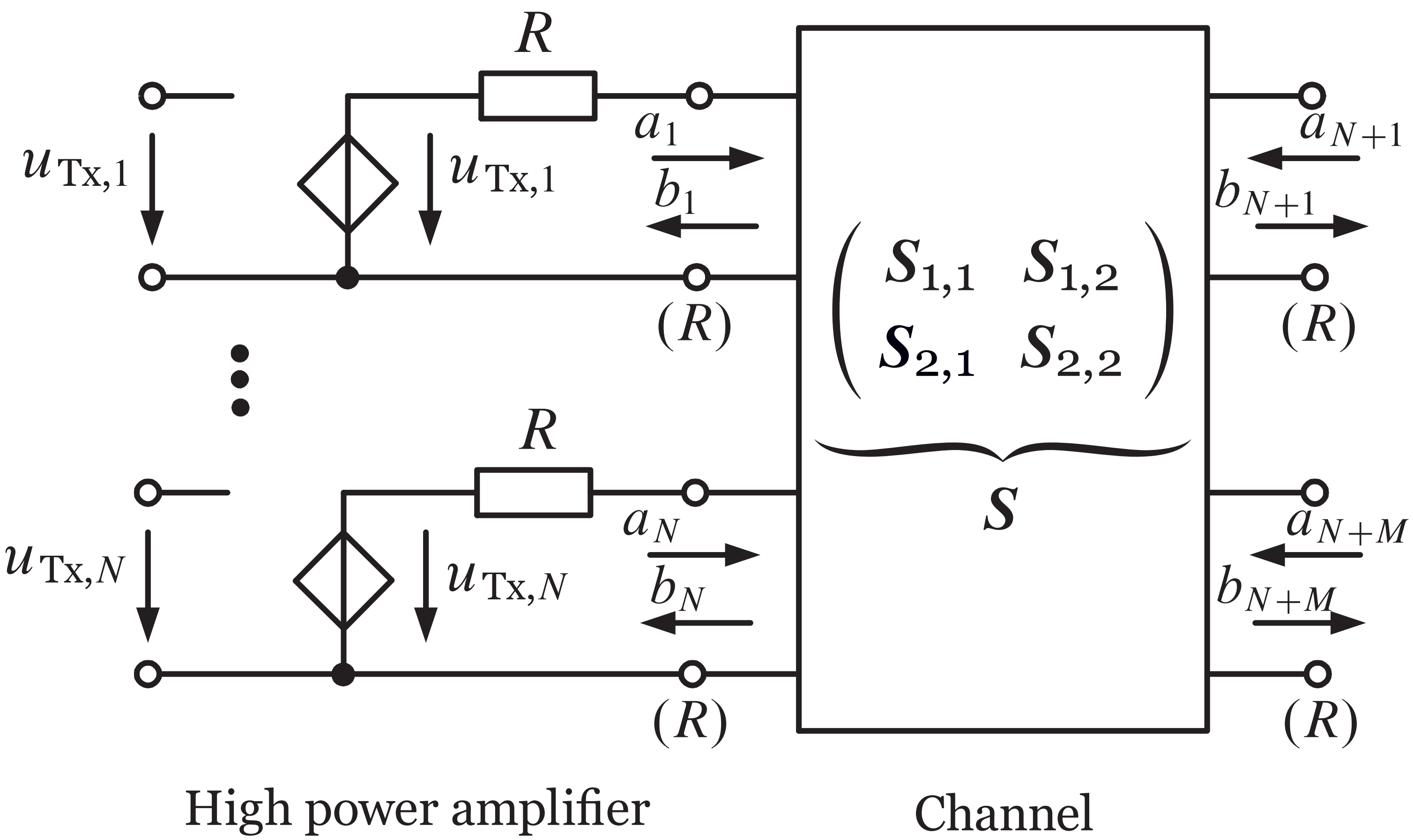
Circuit theoretic system model

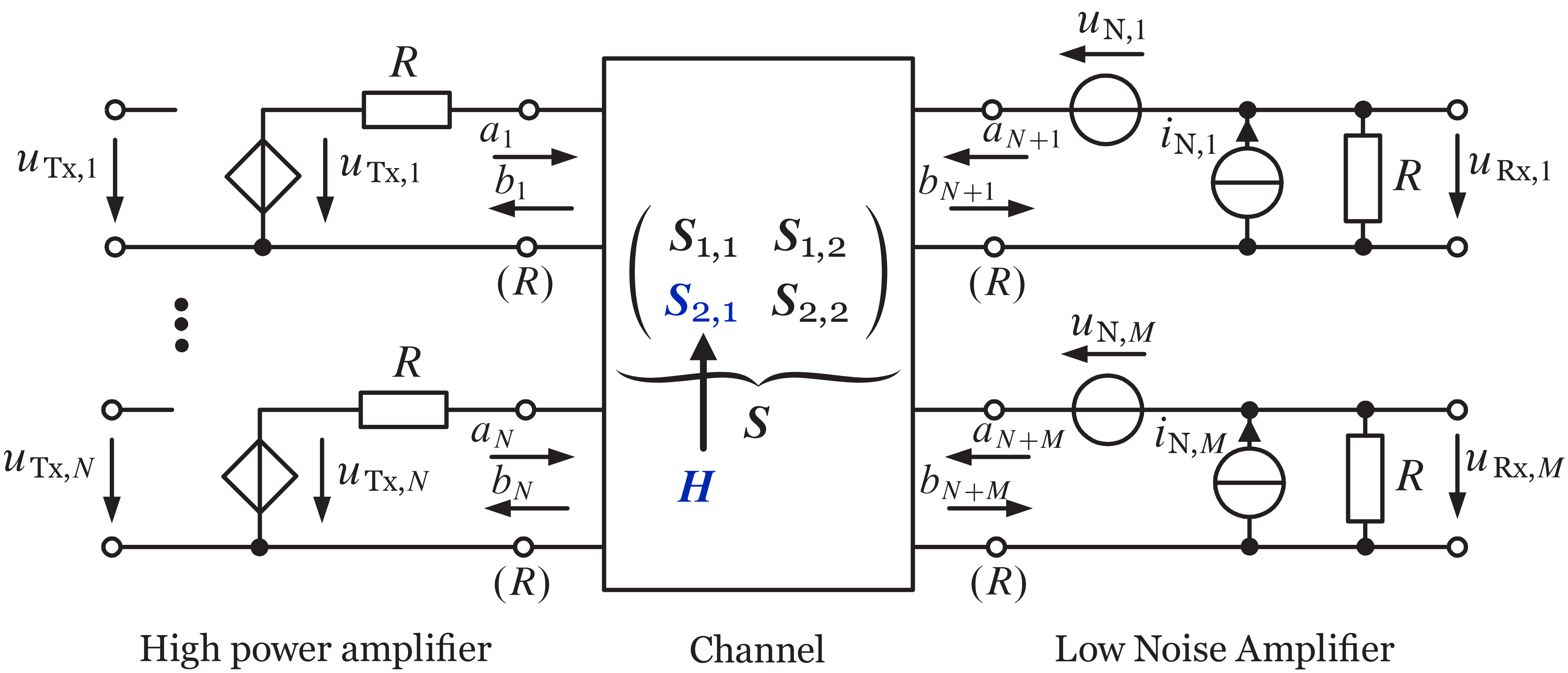


High power amplifier

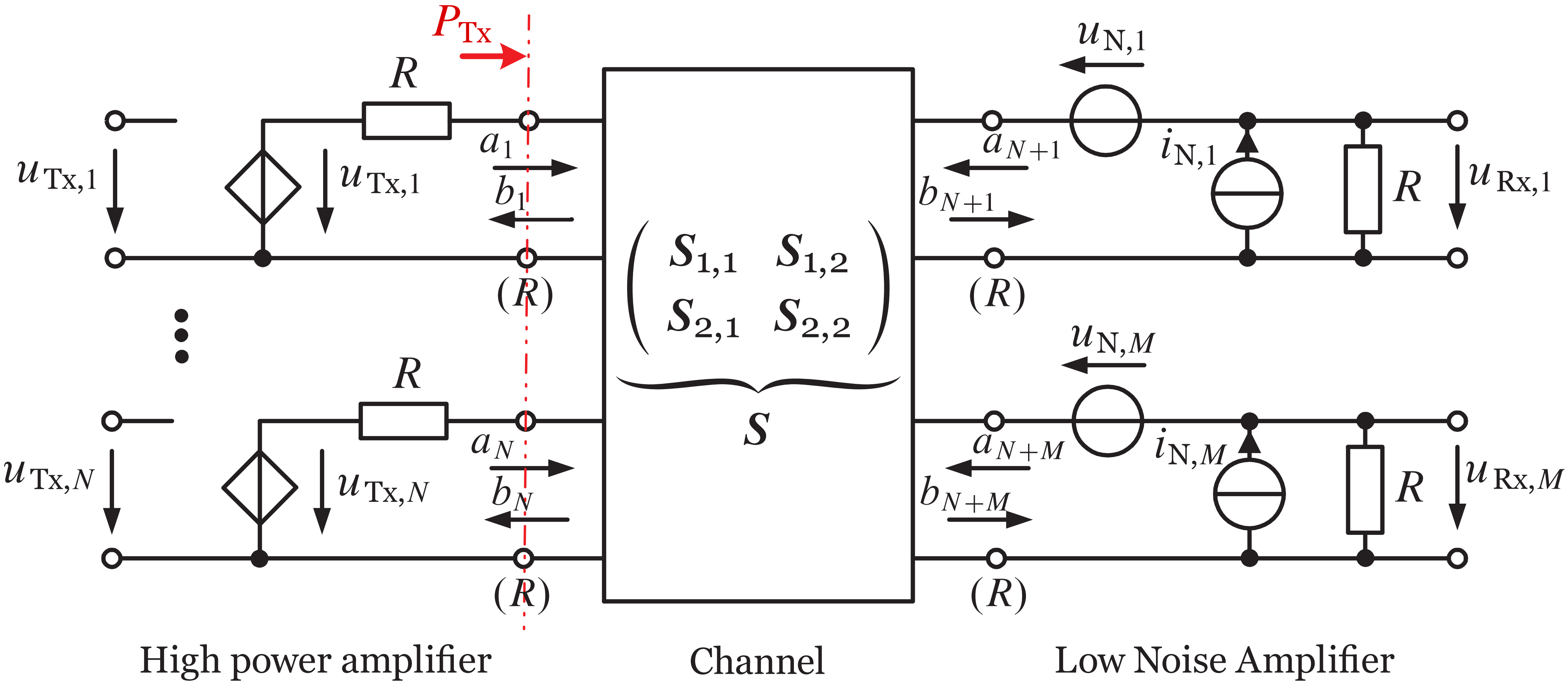


- Scattering matrix: $\boldsymbol{S} \in \mathbb{C}^{(N+M)\times(N+M)}$

Circuit theoretic system model

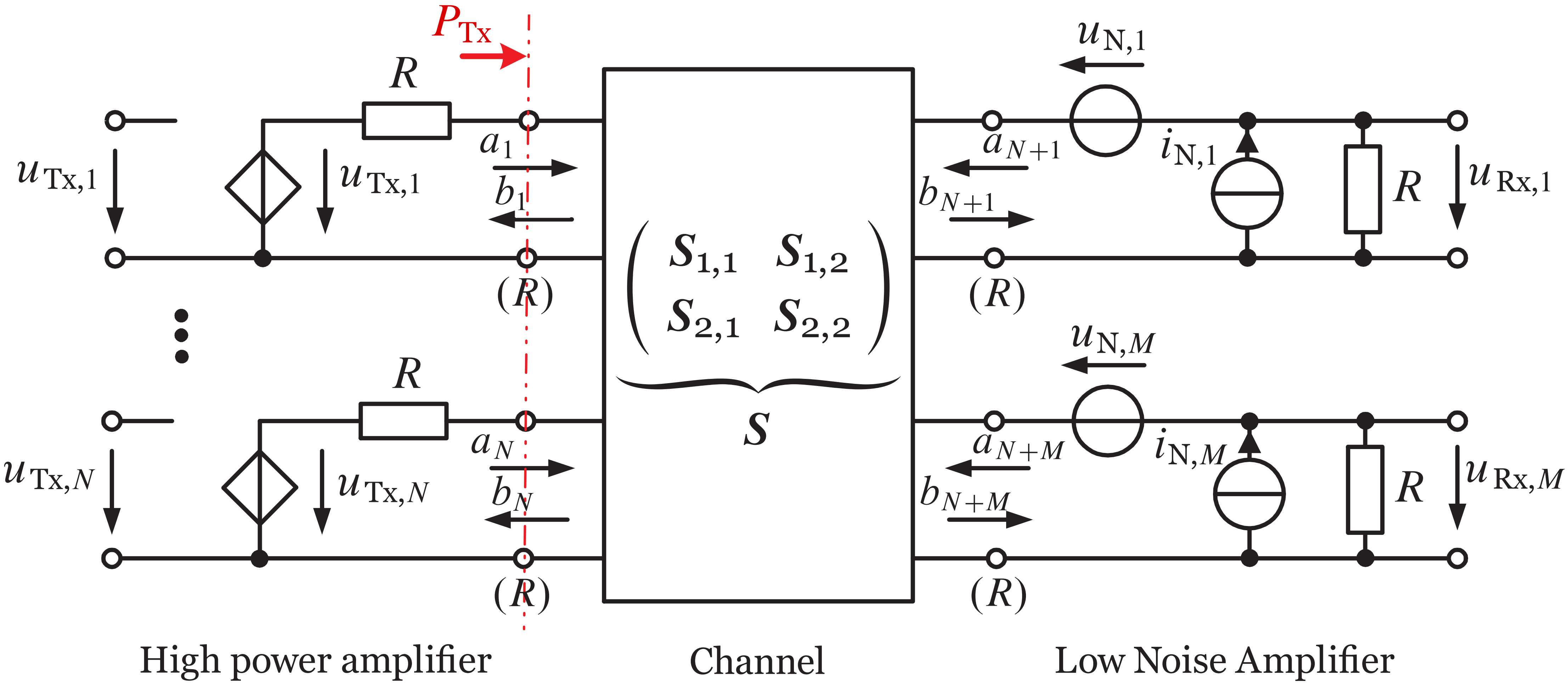


- Scattering matrix: $\boldsymbol{S} \in \mathbb{C}^{(N+M)\times(N+M)}$
- Noise: current noise and voltage noise



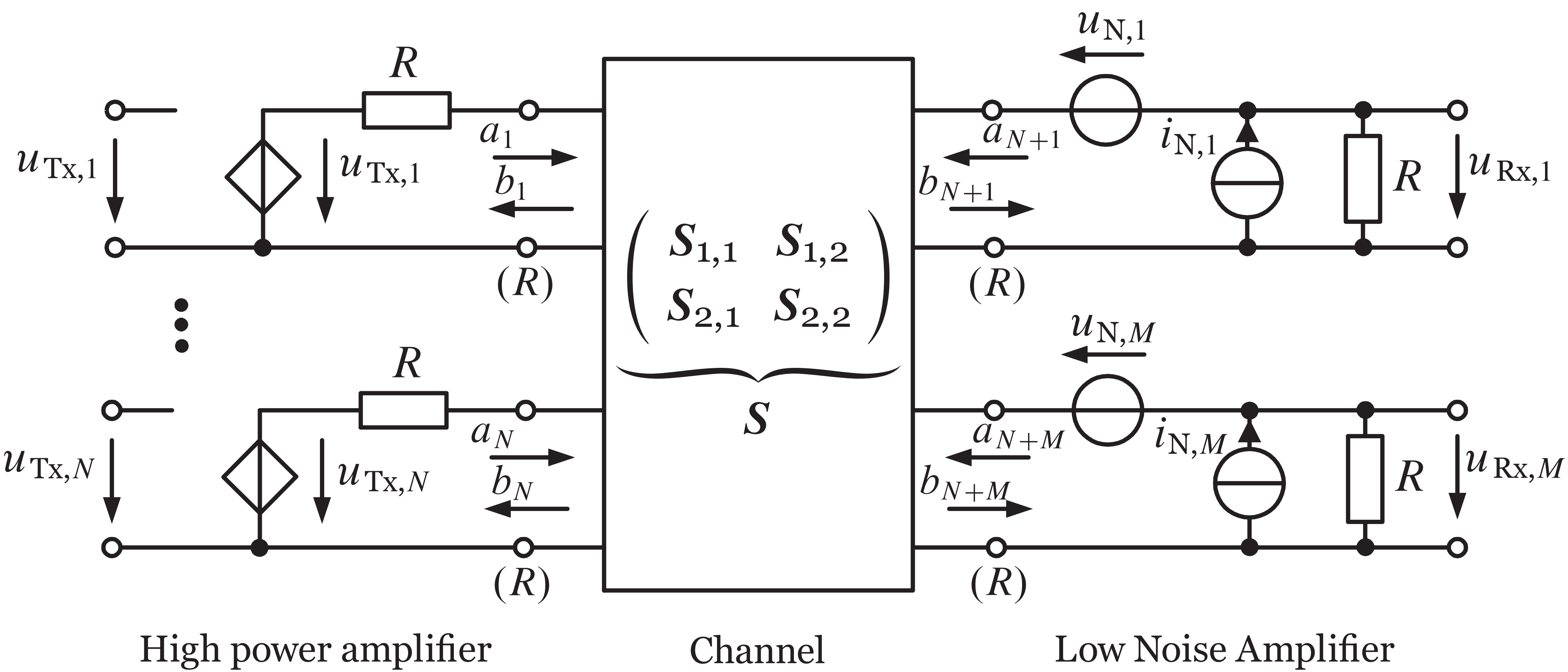
* **Transmit power:** $$P_{\mathrm{Tx}} = \sum_{n=1}^{N} \mathrm{E}\left[|a_n|^2 - |b_n|^2\right]$$ for zero noise

Computing transmit power



* Transmit power:

$$P_{\mathrm{Tx}} = \frac{1}{4R} \mathrm{E}\left[\boldsymbol{u}_{\mathrm{Tx}}^{\mathrm{H}} \boldsymbol{B} \boldsymbol{u}_{\mathrm{Tx}}\right], \quad \boldsymbol{B} = \mathbf{I}_N - \boldsymbol{S}_{1,1}^{\mathrm{H}} \boldsymbol{S}_{1,1}$$



$$\frac{1}{\sqrt{R}}\boldsymbol{u}_{\mathrm{Rx}} = \frac{1}{2\sqrt{R}}\boldsymbol{S}_{2,1}\boldsymbol{u}_{\mathrm{Tx}} + \underbrace{\frac{\sqrt{R}}{2}\left(\mathbf{I}_M + \boldsymbol{S}_{2,2}\right)\boldsymbol{i}_{\mathrm{N}} + \frac{1}{2\sqrt{R}}\left(\mathbf{I}_M - \boldsymbol{S}_{2,2}\right)\boldsymbol{u}_{\mathrm{N}}}_{\text{receiver noise, } \boldsymbol{\eta}}$$

On the receiver noise covariance

- Receiver noise: $\boldsymbol{\eta} = \frac{\sqrt{R}}{2}\left(\mathbf{I}_M + \boldsymbol{S}_{2,2}\right)\boldsymbol{i}_\mathrm{N} + \frac{1}{2\sqrt{R}}\left(\mathbf{I}_M - \boldsymbol{S}_{2,2}\right)\boldsymbol{u}_\mathrm{N}$

- Receiver noise covariance: $\boldsymbol{R}_\eta = \mathrm{E}\left[\boldsymbol{\eta}\boldsymbol{\eta}^\mathrm{H}\right]$

  $\longrightarrow$ Even if $\boldsymbol{u}_\mathrm{N}$, and $\boldsymbol{i}_\mathrm{N}$, are independent, and their components are mutually uncorrelated, the receiver noise is, in general, correlated!

- Example: thermal resistor noise: $\boldsymbol{u}_\mathrm{N} \equiv \mathbf{0}$, $\quad \mathrm{E}\left[\boldsymbol{i}_\mathrm{N}\boldsymbol{i}_\mathrm{N}^\mathrm{H}\right] = 4kTBR^{-1}\mathbf{I}_M$

$$\longrightarrow \quad \boldsymbol{R}_\eta = \frac{R}{4}\left(\mathbf{I}_M + \boldsymbol{S}_{2,2}\right)\mathrm{E}\left[\boldsymbol{i}_\mathrm{N}\boldsymbol{i}_\mathrm{N}^\mathrm{H}\right]\left(\mathbf{I}_M + \boldsymbol{S}_{2,2}\right)^\mathrm{H}$$

$$= kTB \cdot \left(\mathbf{I}_M + \boldsymbol{S}_{2,2}\right)\left(\mathbf{I}_M + \boldsymbol{S}_{2,2}\right)^\mathrm{H}$$

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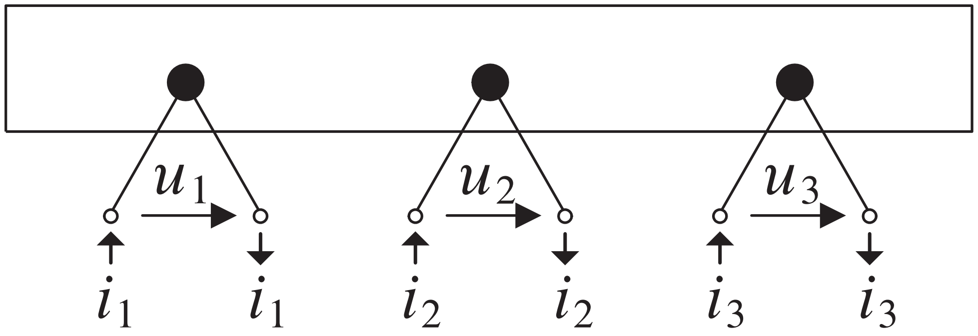
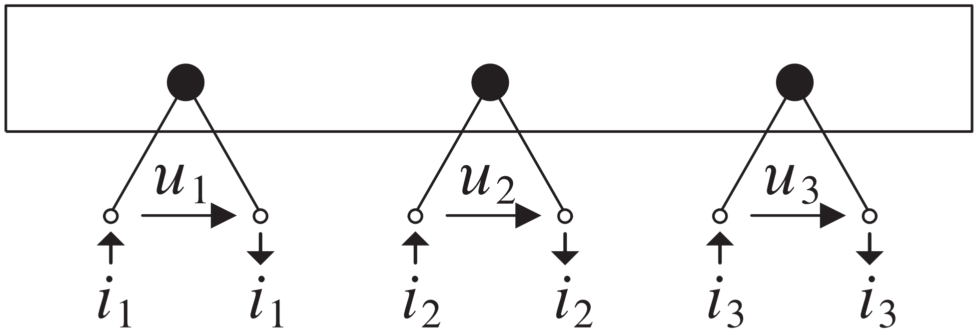
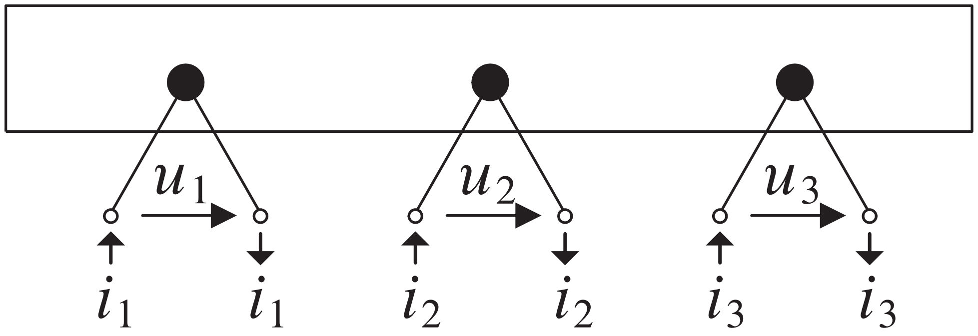

u_1
u_2
u_3
i_1
i_1
i_2
i_2
i_3
i_3



Circuits point of view

1. $\boldsymbol{u} = \boldsymbol{Z}\boldsymbol{i}$
2. $\boldsymbol{Z} = \boldsymbol{Z}^{\mathrm{T}}$ (reciprocity)
3. $P_{\mathrm{in}} = \mathrm{E}\left[\mathrm{Re}\{\boldsymbol{u}^{\mathrm{H}}\boldsymbol{i}\}\right]$

$\longrightarrow \quad P_{\mathrm{in}} = \mathrm{E}\left[\boldsymbol{i}^{\mathrm{H}}\,\mathrm{Re}\{\boldsymbol{Z}\}\,\boldsymbol{i}\right]$



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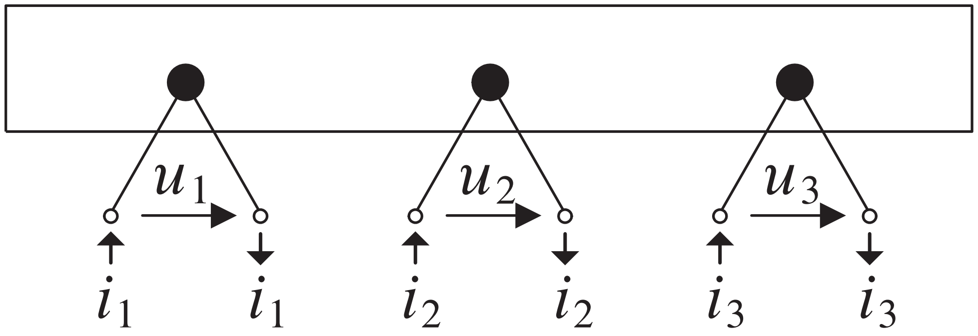
$\longrightarrow \quad P_{\mathrm{in}} = \mathrm{E}\left[\boldsymbol{i}^{\mathrm{H}}\,\mathrm{Re}\{\boldsymbol{Z}\}\,\boldsymbol{i}\right]$

Electro-magnetic view

1. Current $\boldsymbol{i}$, excites $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{H}}$ field
2. Radiated power obtainable from: $P_{\mathrm{rad}} = \mathrm{E}\left[\iint_{\mathrm{sphere}} \mathrm{Re}\left\{\vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}}^{*}\right\} \mathrm{d}\vec{\boldsymbol{A}}\right]$

$\longrightarrow \quad P_{\mathrm{rad}} = Z_0\,\mathrm{E}\left[\boldsymbol{i}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{i}\right]$



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2. $\boldsymbol{Z} = \boldsymbol{Z}^{\mathrm{T}}$ (reciprocity)
3. $P_{\text{in}} = \mathrm{E}\left[\mathrm{Re}\{\boldsymbol{u}^{\mathrm{H}}\boldsymbol{i}\}\right]$

$\longrightarrow \quad P_{\text{in}} = \mathrm{E}\left[\boldsymbol{i}^{\mathrm{H}}\mathrm{Re}\{\boldsymbol{Z}\}\boldsymbol{i}\right]$

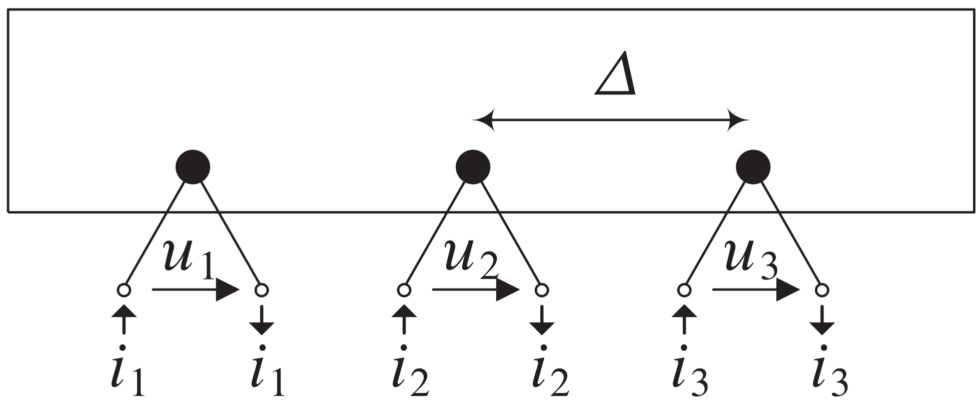
✻ Electro-magnetic view

1. Current $\boldsymbol{i}$, excites $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{H}}$ field
2. Radiated power obtainable from: $P_{\text{rad}} = \mathrm{E}\left[\iint_{\text{sphere}} \mathrm{Re}\left\{\vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}}^*\right\} \mathrm{d}\vec{\boldsymbol{A}}\right]$

$\longrightarrow \quad P_{\text{rad}} = Z_0 \mathrm{E}\left[\boldsymbol{i}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{i}\right]$

✻ Lossless antennas: $P_{\text{rad}} = P_{\text{in}} \longrightarrow \mathrm{Re}\{\boldsymbol{Z}\} = Z_0\boldsymbol{C}$

Uniform linear antenna array of isotrops

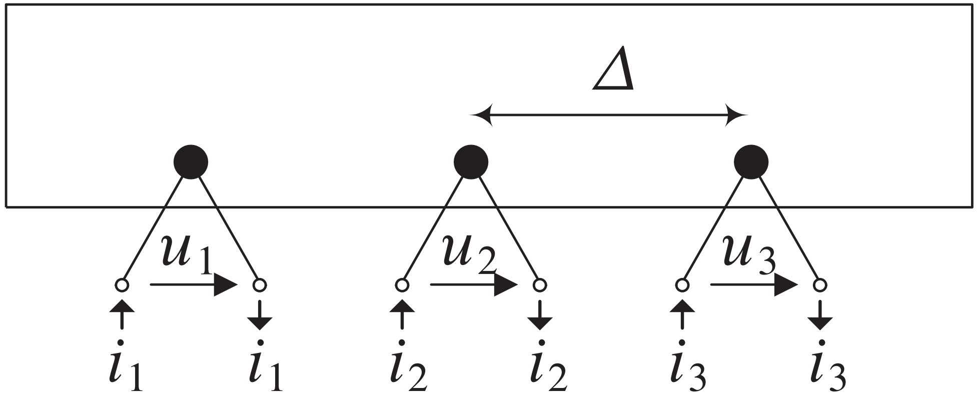


- **Closed-form solution:**

$$(\boldsymbol{C})_{n,m} = \operatorname{sinc}\left(2\pi\frac{\Delta}{\lambda}(m-n)\right)$$

- **Uncoupled antennas:**

$$\frac{\Delta}{\lambda} \in \frac{1}{2}\mathbb{N} \quad \Longleftrightarrow \quad \boldsymbol{C} = \mathbf{I}$$



Impedance matching network:

$$\boldsymbol{Z} = \mathrm{Re}\{\boldsymbol{Z}\} = Z_0 \boldsymbol{C}$$

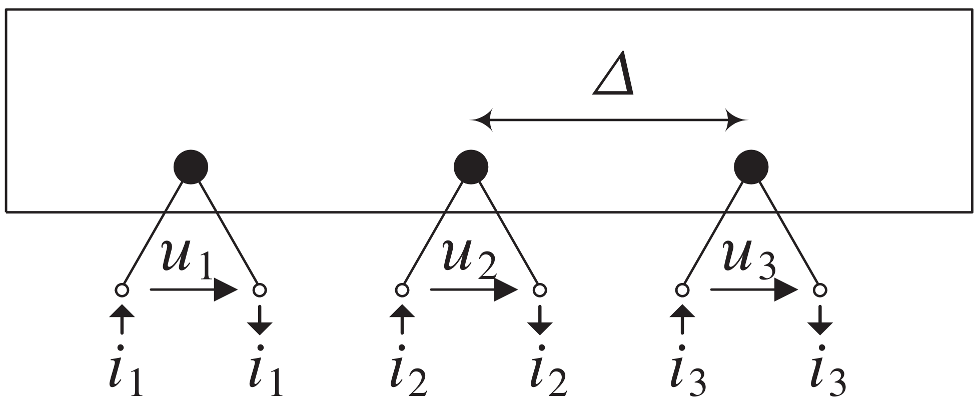
Closed-form solution:

$$(\boldsymbol{C})_{n,m} = \mathrm{sinc}\left(2\pi \frac{\Delta}{\lambda}(m-n)\right)$$

Uncoupled antennas:

$$\frac{\Delta}{\lambda} \in \frac{1}{2}\mathbb{N} \iff \boldsymbol{C} = \mathbf{I}$$

Uniform linear antenna array of isotrops



- Closed-form solution:

$$(\boldsymbol{C})_{n,m} = \operatorname{sinc}\left(2\pi \frac{\Delta}{\lambda}(m-n)\right)$$

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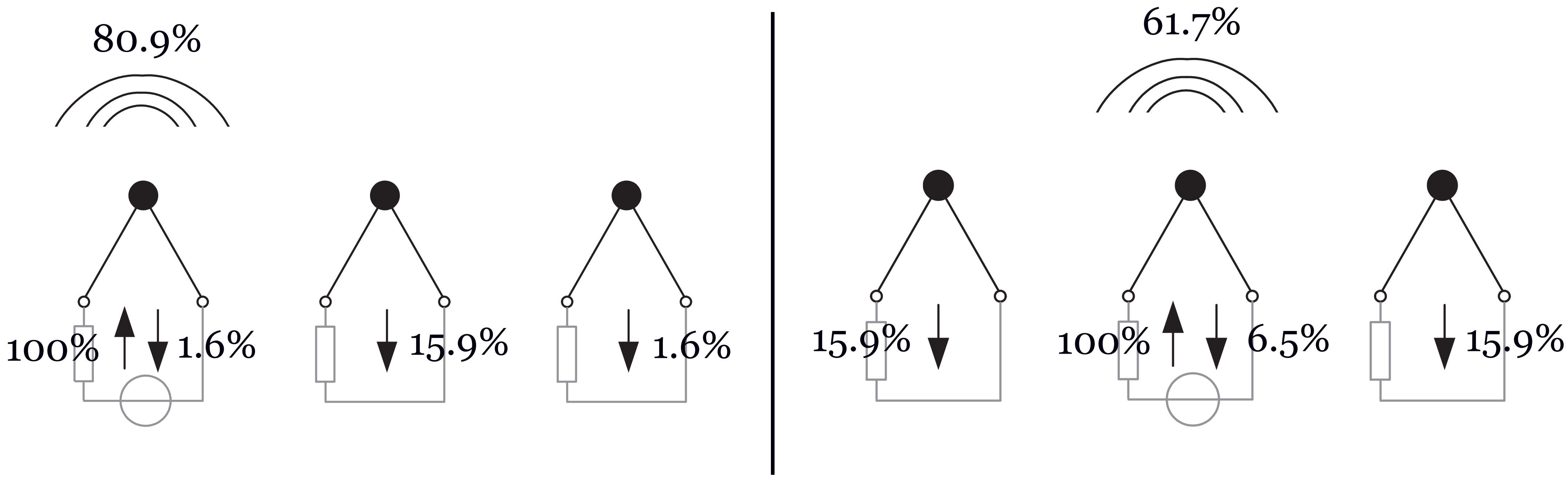
- Impedance matching network:

$$\boldsymbol{Z} = \operatorname{Re}\{\boldsymbol{Z}\} = Z_0 \boldsymbol{C}$$

- Scattering Matrix: $$\boldsymbol{S}_{i,i} = \left(\boldsymbol{C} + \frac{R}{Z_0}\mathbf{I}\right)^{-1}\left(\boldsymbol{C} - \frac{R}{Z_0}\mathbf{I}\right), \quad i \in \{1,2\}$$

- $N = 3, \;\; \Delta = \lambda/4, \;\;$ and $\;\; R = Z_0$

$$\boldsymbol{S}_{1,1} = \frac{1}{\pi^2 - 2}\begin{pmatrix} -1 & \pi & -1 \\ \pi & -2 & \pi \\ -1 & \pi & -1 \end{pmatrix}$$



- The $\boldsymbol{B}$–Matrix: $\boldsymbol{B} = \mathbf{I}_N - \boldsymbol{S}_{1,1}^{\mathrm{H}} \boldsymbol{S}_{1,1} \quad \Longrightarrow \quad P_{\mathrm{Tx}} = \frac{1}{4R} \mathrm{E}\left[\boldsymbol{u}_{\mathrm{Tx}}^{\mathrm{H}} \boldsymbol{B} \boldsymbol{u}_{\mathrm{Tx}}\right]$

$$\boldsymbol{B} = \frac{4R}{Z_0} \left(\boldsymbol{C}_{\mathrm{Tx}} + \frac{R}{Z_0} \mathbf{I}_N \right)^{-1} \boldsymbol{C}_{\mathrm{Tx}} \left(\boldsymbol{C}_{\mathrm{Tx}} + \frac{R}{Z_0} \mathbf{I}_N \right)^{-1}$$

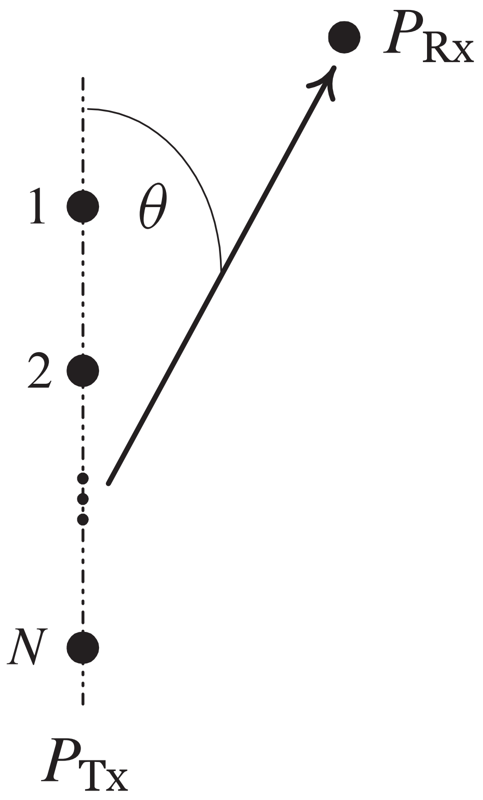
- Thermal noise covariance: $\boldsymbol{R}_\eta = \sigma^2 \cdot \left(\mathbf{I}_M + \boldsymbol{S}_{2,2} \right) \left(\mathbf{I}_M + \boldsymbol{S}_{2,2} \right)^{\mathrm{H}}$

$$\sigma^2 = kTB$$

$$\boldsymbol{R}_\eta = 4\sigma^2 \cdot \left(\boldsymbol{C}_{\mathrm{Rx}} + \frac{R}{Z_0} \mathbf{I}_M \right)^{-1} \boldsymbol{C}_{\mathrm{Rx}}^2 \left(\boldsymbol{C}_{\mathrm{Rx}} + \frac{R}{Z_0} \mathbf{I}_M \right)^{-1}$$

Contents

Transmit array gain 1/2



- **Optimum transmit beamforming:**

  $\longrightarrow$ Maximize SNR at receiver for a given P_{Tx}

- **Transmit array gain:**

$$A_{\mathrm{Tx}} \overset{\mathrm{def}}{=} \frac{\mathrm{SNR}^{\max}}{\mathrm{SNR}^{\mathrm{iso}}} = \frac{P_{\mathrm{Rx}}^{\max}}{P_{\mathrm{Rx}}^{\mathrm{iso}}} = N \frac{\boldsymbol{\alpha}^{\mathrm{H}}(\theta)\boldsymbol{C}_{\mathrm{Tx}}^{-1}\boldsymbol{\alpha}(\theta)}{\boldsymbol{\alpha}^{\mathrm{H}}(\theta)\boldsymbol{\alpha}(\theta)}$$

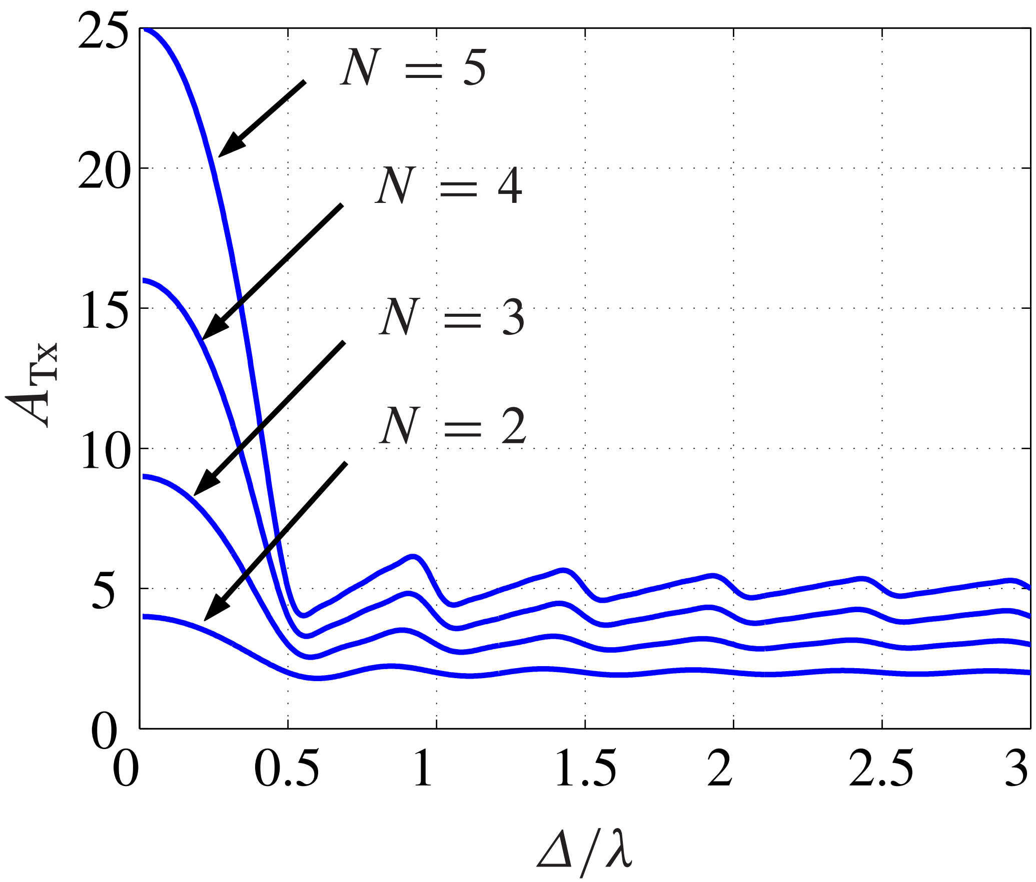
$$\boldsymbol{\alpha}(\theta) = \begin{pmatrix} 1 & \mathrm{e}^{-\mathrm{j}\xi} & \mathrm{e}^{-2\mathrm{j}\xi} & \dots & \mathrm{e}^{-(N-1)\mathrm{j}\xi} \end{pmatrix}^{\mathrm{T}}$$

$$\xi = 2\pi \frac{\Delta}{\lambda} \cos\theta$$

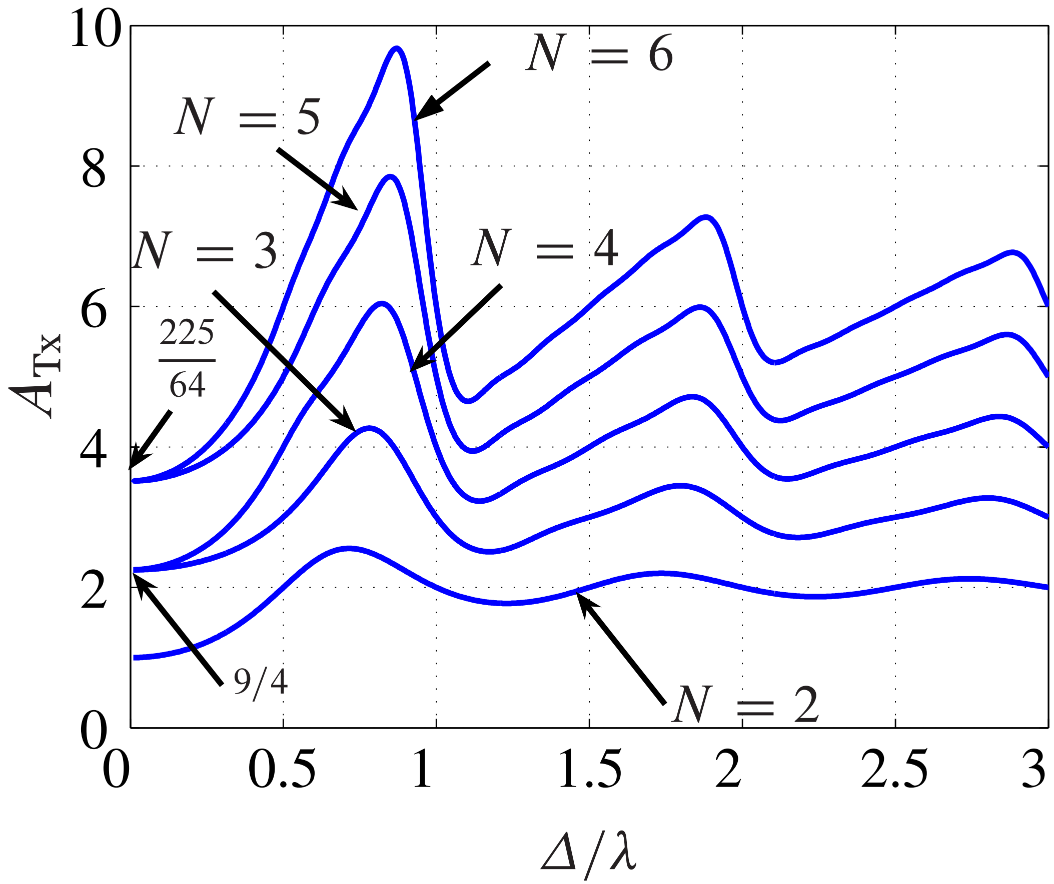
- **Transmit array gain depends on:**
  1. number of antennas (of course)
  2. antenna spacing
  3. direction of beamforming

Transmit array gain 2/2

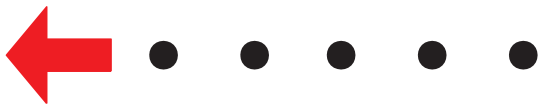
✻ »End-Fire«

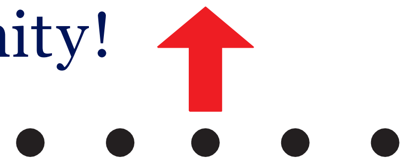


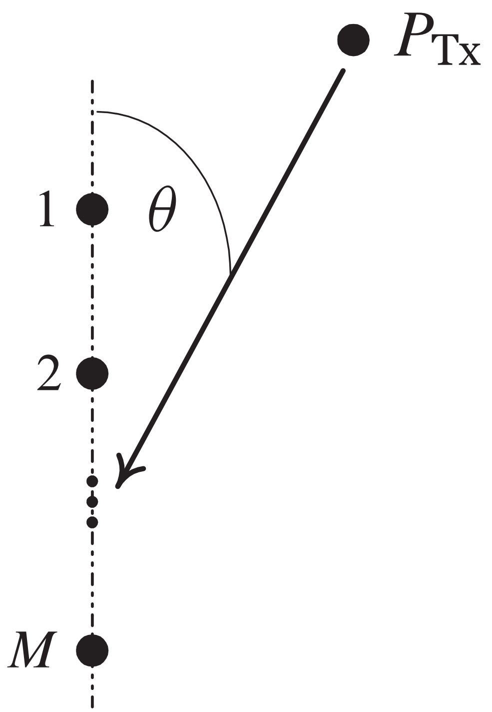
✻ »Front-Fire«



… known in the antenna community!







- **Optimum receive beamforming:**

  $\longrightarrow$ Maximize SNR for a given P_{Tx}

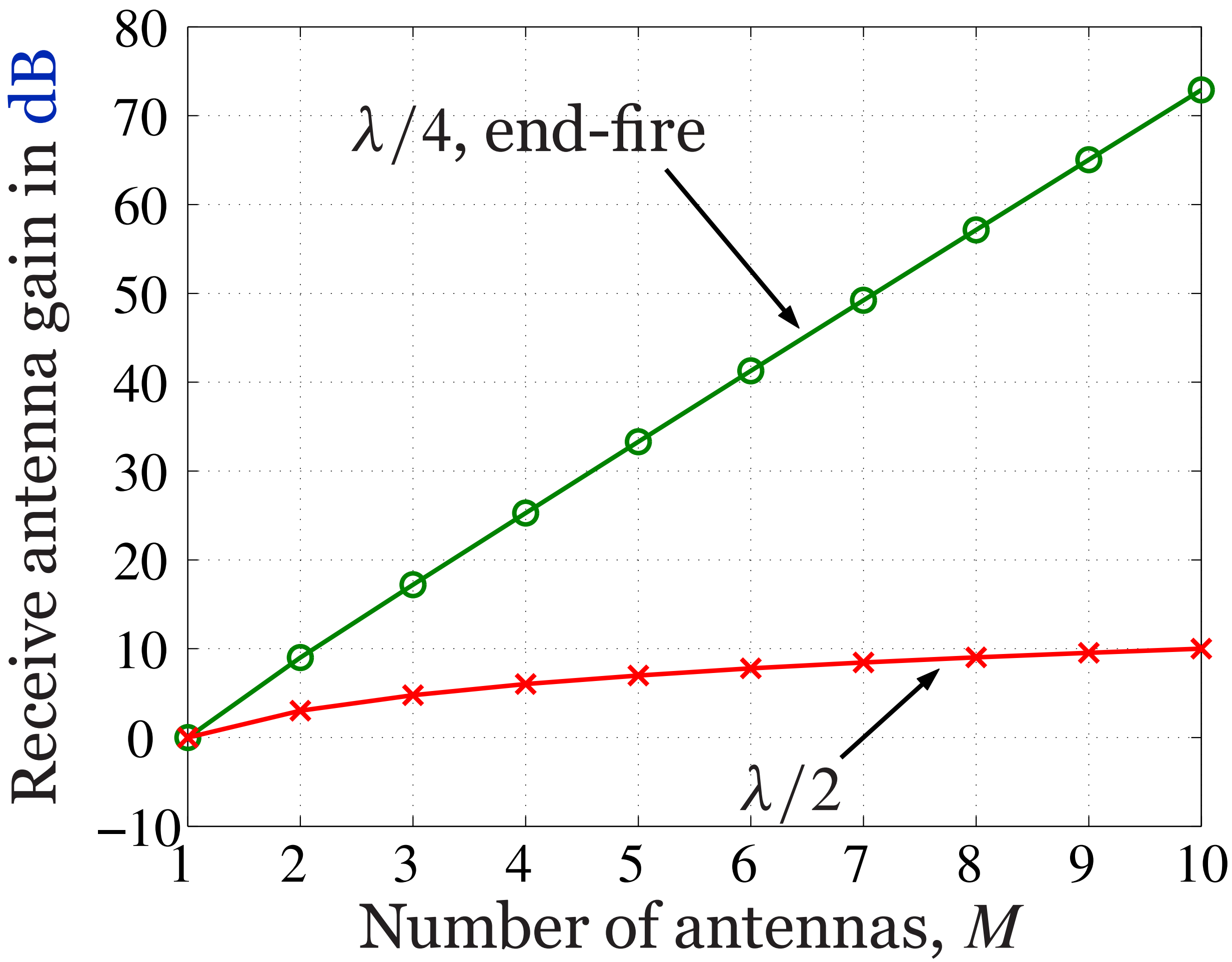
- **Receive array gain:**

$$A_{\mathrm{Rx}} \overset{\mathrm{def}}{=} \frac{\mathrm{SNR}^{\mathrm{max}}}{\mathrm{SNR}^{\mathrm{iso}}} = M \frac{\boldsymbol{\alpha}^{\mathrm{H}}(\theta)\boldsymbol{C}_{\mathrm{Rx}}^{-2}\boldsymbol{\alpha}(\theta)}{\boldsymbol{\alpha}^{\mathrm{H}}(\theta)\boldsymbol{\alpha}(\theta)}$$

$$\boldsymbol{\alpha}(\theta) = \begin{pmatrix} 1 & \mathrm{e}^{-\mathrm{j}\xi} & \mathrm{e}^{-2\mathrm{j}\xi} & \dots & \mathrm{e}^{-(M-1)\mathrm{j}\xi} \end{pmatrix}^{\mathrm{T}}$$

$$\xi = 2\pi \frac{\Delta}{\lambda} \cos\theta$$

- **Receive array gain depends on:**
  1. number of antennas (of course)
  2. antenna spacing
  3. direction of beamforming

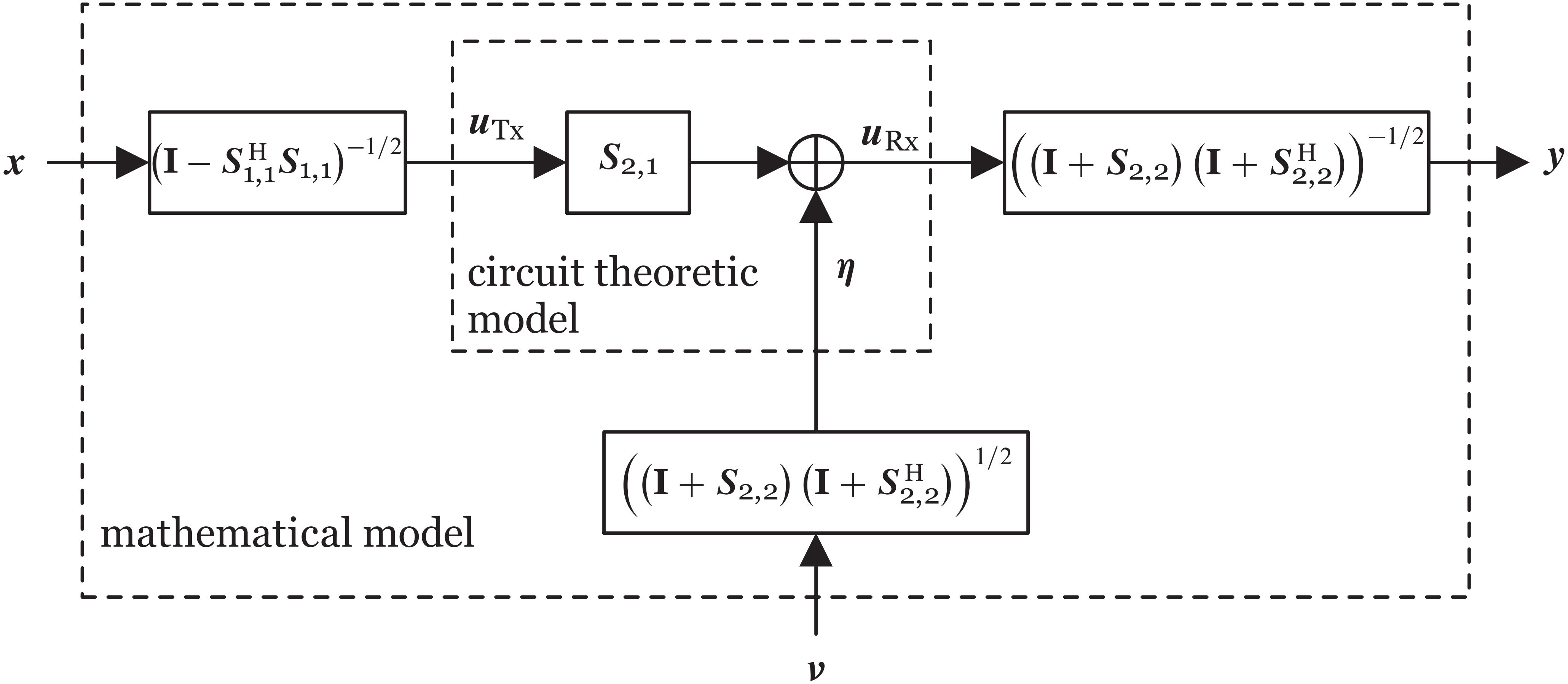


… to our best knowledge,
such a result is not available in the published literature!

Contents

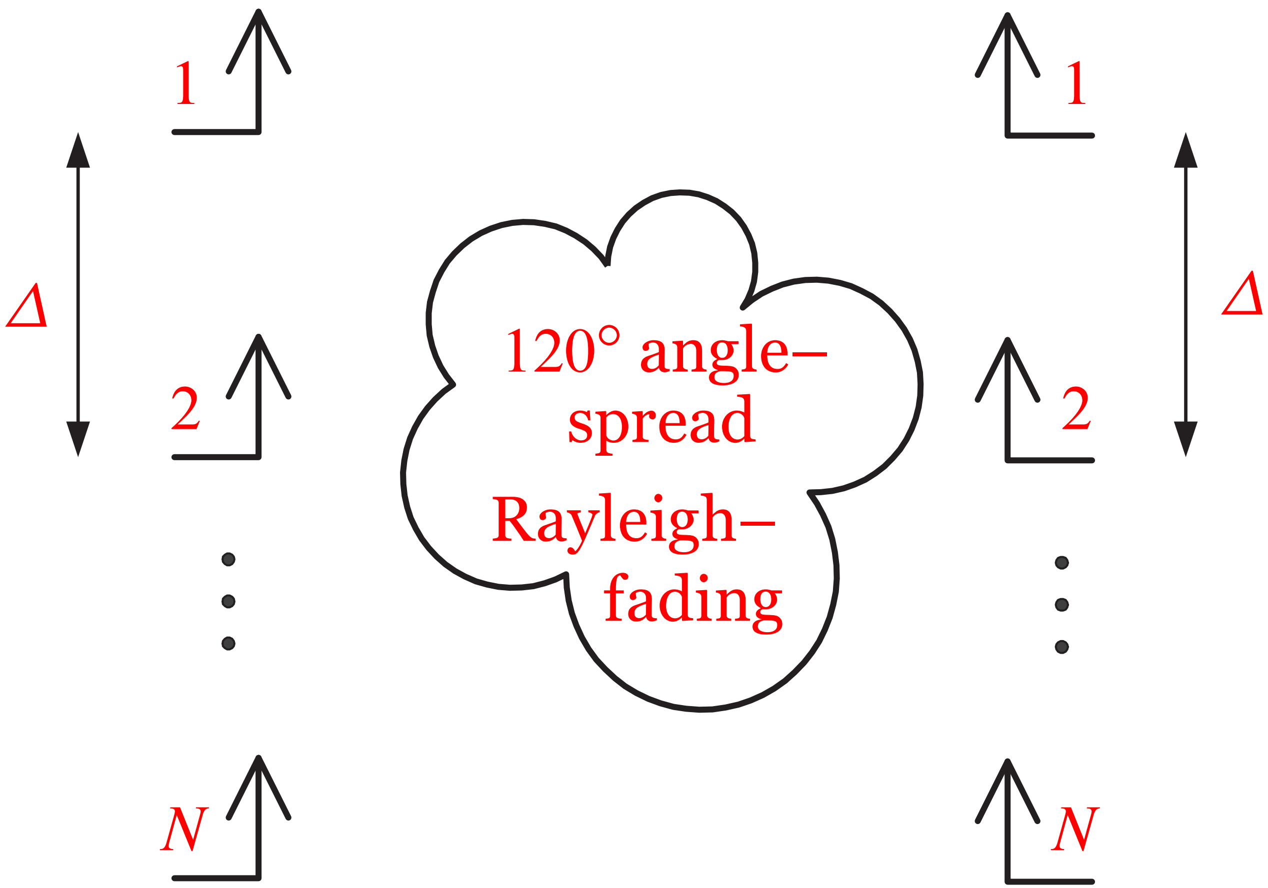
$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}$$

- Channel input $\boldsymbol{x} = \frac{1}{2\sqrt{R}} \left(\mathbf{I}_N - \boldsymbol{S}_{1,1}^{\mathrm{H}}\boldsymbol{S}_{1,1}\right)^{1/2} \boldsymbol{u}_{\mathrm{Tx}}$
- Channel output $\boldsymbol{y} = \frac{1}{\sqrt{R}} \left(\left(\mathbf{I}_M + \boldsymbol{S}_{2,2}\right)\left(\mathbf{I}_M + \boldsymbol{S}_{2,2}^{\mathrm{H}}\right)\right)^{-1/2} \boldsymbol{u}_{\mathrm{Rx}}$
- Transmission: $\boldsymbol{H} = \left(\left(\mathbf{I}_M + \boldsymbol{S}_{2,2}\right)\left(\mathbf{I}_M + \boldsymbol{S}_{2,2}^{\mathrm{H}}\right)\right)^{-1/2} \boldsymbol{S}_{2,1} \left(\mathbf{I}_N - \boldsymbol{S}_{1,1}^{\mathrm{H}}\boldsymbol{S}_{1,1}\right)^{-1/2}$
- Noise model: $\boldsymbol{R}_v = \sigma^2 \mathbf{I}_M$
- Transmit power model: $P_{\mathrm{Tx}} = \mathrm{E}\left[||\boldsymbol{x}||_2^2\right]$

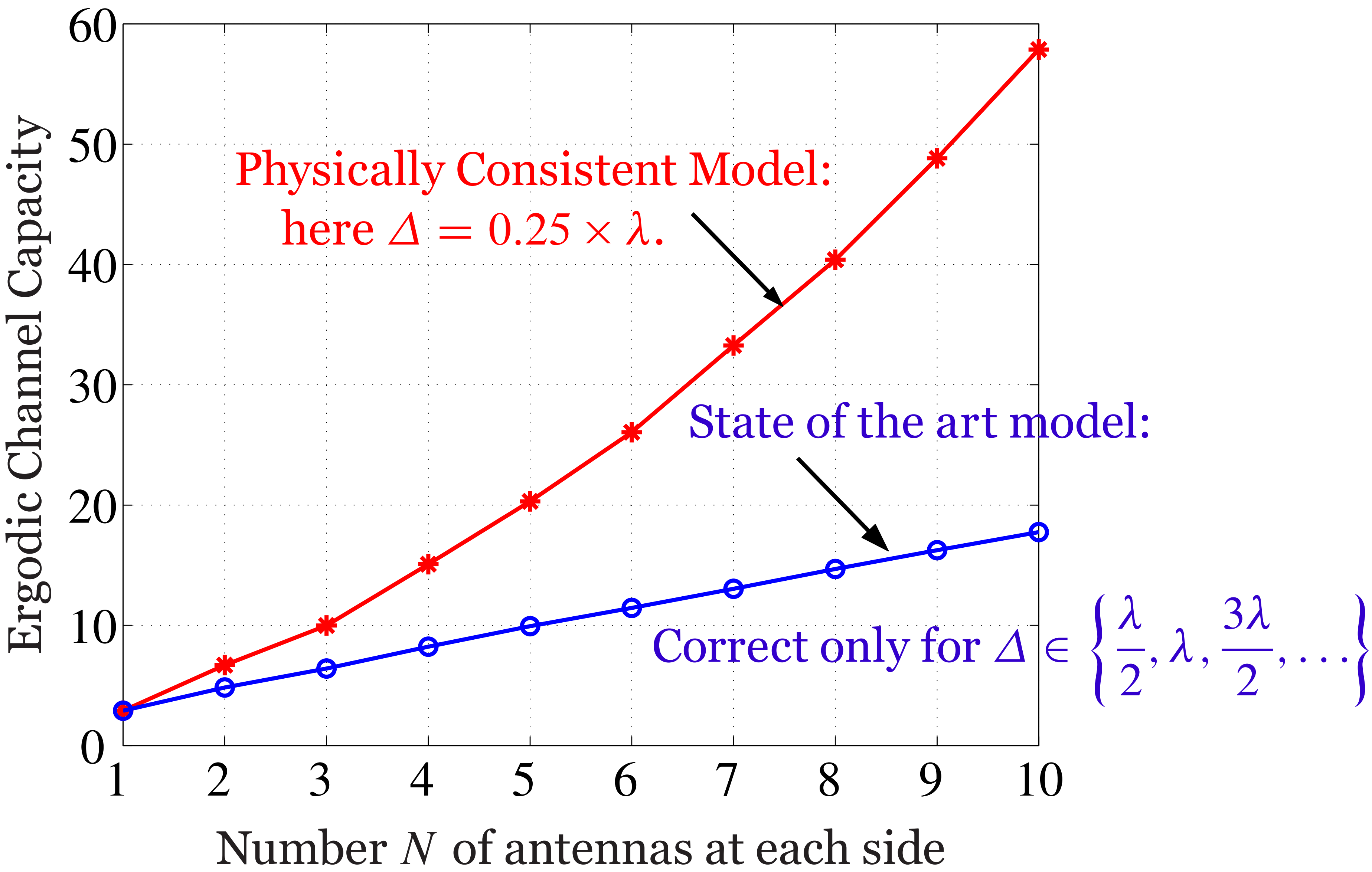

x
$\left(\mathbf{I} - S_{1,1}^{\mathrm{H}} S_{1,1}\right)^{-1/2}$
u_{Tx}
$S_{2,1}$
u_{Rx}
$\left(\left(\mathbf{I} + S_{2,2}\right)\left(\mathbf{I} + S_{2,2}^{\mathrm{H}}\right)\right)^{-1/2}$
y
circuit theoretic model
η
$\left(\left(\mathbf{I} + S_{2,2}\right)\left(\mathbf{I} + S_{2,2}^{\mathrm{H}}\right)\right)^{1/2}$
mathematical model
v

Example: Symmetric MIMO System

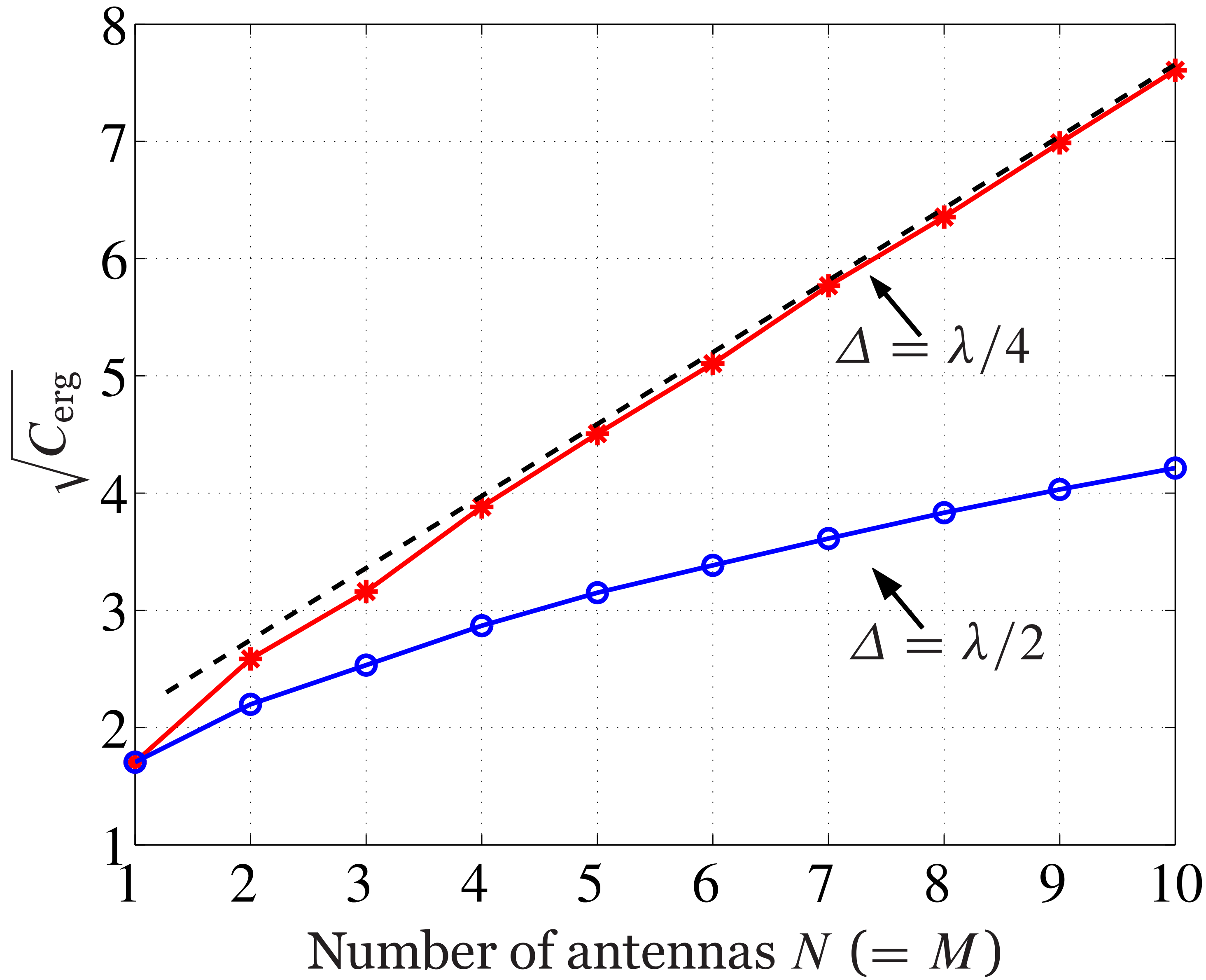
- Consider a $N \times N$ – MIMO system
- Uniform Linear Array (ULA) with isotropic radiators
- Thermal Noise



Ergodic Channel Capacity



Square Root of Ergodic Channel Capacity



Contents

Some Open Problems

- Sensitivity (and what to do about it)
- Design of Matching Networks
- Design of Low-noise Amplifiers
- Optimizing Antenna Characteristics
- ...

Conclusion

Now after all do communication engineers need circuit theory?

- No, if they are not aiming at a complete understanding of the multiport communication system and if they are satisfied with non optimum solutions.
- Yes, they do, if they are aiming at an optimum solution.

Therefore, we should work on complementing the mathematical theory of communication with a circuit theory of communication!

Josef A. Nossek

DO COMMUNICATION ENGINEERS NEED CIRCUIT THEORY?

Taipei, Taiwan, 25. May 2009

Thank you!